

# Endogenous Depth of Reasoning\*

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## Abstract

We introduce a model of strategic thinking in games of initial response. Unlike standard models of strategic thinking, in this framework the player’s ‘depth of reasoning’ is endogenously determined, and it can be disentangled from his beliefs over his opponent’s cognitive bound. In our approach, individuals act as if they follow a cost-benefit analysis. The depth of reasoning is a function of the player’s cognitive abilities and his payoffs. The costs are exogenous and represent the game theoretical sophistication of the player; the benefit instead is related to the game payoffs. Behavior is in turn determined by the individual’s depth of reasoning and his beliefs about the reasoning process of the opponent. Thus, in our framework, payoffs not only affect individual choices in the traditional sense, but they also shape the cognitive process itself. Our model delivers testable implications on players’ chosen actions as incentives and opponents change. We then test the model’s predictions with an experiment. We administer different treatments that vary beliefs over payoffs and opponents, as well as beliefs over opponents’ beliefs. The results of this experiment, which are not accounted for by current models of reasoning in games, strongly support our theory. We also show that the predictions of our model are highly consistent, both qualitatively and quantitatively, with well-known unresolved empirical puzzles. Our approach therefore serves as a novel, unifying framework of strategic thinking that allows for predictions across games.

**Keywords:** cognitive cost – depth of reasoning – higher-order beliefs – level-k reasoning – strategic thinking – theory of mind

**JEL Codes:** C72; C92; D80.

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# 1 Introduction

The relevance of economic incentives to individual behavior has long been recognized, but little is known about the effect of incentives on individuals' reasoning processes in strategic settings. The vast experimental literature on initial responses in games shows that individuals' choices depart systematically from classical equilibrium predictions, and the observed regularities suggest that individuals follow distinct, stepwise reasoning procedures, of which they perform only a few steps.<sup>1</sup> But the step reached, or *depth of reasoning*, may depend on the stakes of the game. Furthermore, beliefs over opponents may also affect choices, and the strategic sophistication according to which individuals play need not coincide with their actual sophistication. Hence, depth of reasoning may vary across strategic settings, and interpreting observed behavior as a measure of cognitive ability is subject to an endogeneity problem. Accounting for the interaction of cognitive abilities, incentives and beliefs in strategic settings is therefore key to understanding cognition and improving the predictive power of game theory.

In this paper we introduce a framework in which players' depth of reasoning is endogenously determined by a procedure that relates individuals' cognitive abilities to the payoffs of the game. Behavior in turn follows from the individual's depth of reasoning and his beliefs about the reasoning process of the opponent. Thus, in our approach, payoffs not only affect individual choices in the traditional sense, but they also shape the cognitive process itself. We next present an experimental test of our theory. The experimental results reveal that individuals change their behavior in a systematic way as payoffs and opponents change, thereby confirming that an endogeneity problem is present when players' cognitive bounds are assessed from their behavior in isolated games. Moreover, these findings are consistent with the predictions of our theory. To further demonstrate the reach of our approach, we then add structure to our baseline model and demonstrate that it explains well-known empirical puzzles. In particular, we consider Goeree and Holt's (2001) influential "Ten Little Treasures and Ten Intuitive Contradictions" paper, and show that our model is highly consistent with their results, both qualitatively and quantitatively. This analysis also serves to show how our model can be used to make inferences and sharp predictions that hold across different games.

The fundamental feature of our framework is that players act *as if* they weigh the incremental value of additional rounds of reasoning against an incremental cost of learning more about the game from introspection.<sup>2</sup> While the cognitive cost is exogenous, the 'value of reasoning' is connected to the game payoffs. In this model, increasing the stakes of the game

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<sup>1</sup>For a recent survey on the empirical and theoretical literature on strategic thinking see Crawford, Costa-Gomes and Iriberry (2012). Particularly important within this area is the literature on level- $k$  reasoning, first introduced by Nagel (1995) and Stahl and Wilson (1994, 1995). Camerer, Ho and Chong (2004) propose the closely related 'cognitive hierarchy' model, in which level- $k$  types respond to a distribution of lower types, and Goeree and Holt (2004) introduce noise in the reasoning process. Level- $k$  models have been extended to study communication (Crawford, 2003), incomplete information (Crawford and Iriberry, 2007) and other games. For recent theoretical work inspired by these ideas, see Strzalecki (2014), Kets (2012) and Kneeland (2014).

<sup>2</sup>The as-if approach, in which the cost-benefit analysis need not be viewed as being performed consciously, circumvents the infinite regress problem in which it would be costly to think about how to determine the value of reasoning, which itself is costly, and so forth (see Lipman (1991)). In Alaoui and Penta (2015) we pursue an axiomatic approach to players' reasoning, in which the cost-benefit analysis emerges as a representation.

provides individuals with stronger incentives to reason, which may induce them to perform more rounds of reasoning. But depth of reasoning need not coincide with the sophistication of the chosen action. When facing opponents that they perceive to be more sophisticated than themselves, subjects play according to their own cognitive bound. But when facing less sophisticated opponents, they play according to less rounds of reasoning. We note that the notion of playing a more sophisticated opponent is natural in this setting, thereby resolving a well-known conceptual difficulty of the level- $k$  approach. Other predictions of the model relate a player's choice and depth of reasoning to his and his opponents' incentives to reason, his beliefs about the opponents' cognitive abilities, and to higher-order beliefs.

A cost-benefit approach to modeling the reasoning process presents several advantages. In addition to holding intuitive appeal, this approach bridges the study of strategic thinking with standard economic concepts. It also provides a tractable way of analyzing complex interactions, since the comparative statics as incentives to reason, beliefs and higher-order beliefs change can be decomposed into shifts of the benefit and cost functions. But as this is an unconventional domain of analysis, the extent to which this approach is useful or empirically relevant is not clear. Investigating its empirical relevance, however, presents one important difficulty: since there is no a priori obvious way of specifying the costs and value of reasoning, it is crucial to isolate the core predictions of the approach, which hold independently of the assumptions on the specific functional forms. For this reason, we first introduce a general framework, with minimal restrictions on the cost and value of reasoning.<sup>3</sup> Using this 'detail-free' model, we focus on the interaction between players' incentives to reason, their beliefs and their higher-order beliefs, and show that this model delivers a rich set of testable predictions. The detail-free model therefore provides a coherent and tractable framework for analyzing the complex interaction between the distinct forces at play, and provides the necessary guidance for the design of an empirical test of the core predictions of the cost-benefit approach.

We discuss next our experimental design. Besides testing the predictions of the detail-free model, the experiment also serves the broader purpose of documenting whether players' steps of reasoning vary systematically as their incentives and beliefs over their opponents' cognitive abilities change.<sup>4</sup> We consider two different ways of changing the agents' beliefs. In both cases, we divide the subjects into two groups whose labels are perceived to be informative about game theoretic sophistication. In the first case, we separate the subjects into two groups by degrees of study. In the second, subjects take a test of our design, and are then separated by their score, which can either be 'high' or 'low'. We then use these labels to vary agents' beliefs. These changes serve to test the model's predictions that agents play according to a lower depth of reasoning when playing against opponents they take to be less sophisticated. Our theory also allows players to not only take into account the (perceived) sophistication of the opponent, but

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<sup>3</sup>Recent work by Choi (2012) also incorporates a cost-benefit approach in a setting of strategic thinking and learning in networks. We discuss the connections with that paper and other related models in Section 2.4.

<sup>4</sup>A recent experiment by Agranov, Potamites, Schotter and Tergiman (2012) makes the simple but important point that beliefs do change the average number of rounds performed in a standard beauty contest. Palacios-Huerta and Volij (2009) explore a related point in the dynamic context of the centipede game.

also the opponent’s belief over the player’s own sophistication. To account for these higher-order beliefs effects, we administer treatments in which subjects classified under a label play against the action that subjects from the other label have played against each other.

To test whether players respond to increased incentives of doing more rounds of introspection in the manner that is predicted by our model, we increase the stakes of the baseline game. We then compare the distributions of the chosen actions across these different treatments. Our results are consistent with the prediction that subjects play according to more rounds of introspection when stakes are increased and when opponents are believed to be more sophisticated. The results are also in line with the predictions over higher-order beliefs effects. Overall, these results show that individuals change their behavior in a systematic manner that is not endogenized by existing models of strategic reasoning, but that is strongly consistent with our theoretical predictions. These findings therefore establish the importance of accounting for the endogeneity of the depth of reasoning and support the validity of our general approach.

Lastly, we add structure to the model and use it to derive sharp predictions that hold across games. In particular, we consider the games in the influential paper by Goeree and Holt (2001). Goeree and Holt’s findings are intuitive, but difficult to reconcile with standard models. Nonetheless, we show that our theory does not only fit the qualitative results, it also performs well from a quantitative viewpoint. Using a specification of the model with a single free parameter, we calibrate it to match the data in one of Goeree and Holt’s games, and derive predictions for the others. We find that these predictions are highly consistent with Goeree and Holt’s empirical results. Since the ‘little treasures’ are very different from one another, ranging from Basu’s (1994) traveler’s dilemma to matching pennies and coordination games, these findings show that our model applies to a broad spectrum of games. From a methodological viewpoint, these results further confirm that, by shifting the focus of the analysis to the comparative statics, our theory allows inferences and predictions that hold across games, and uncovers the deeper mechanisms of strategic thinking.

## 2 Theory

We begin by describing players’ reasoning process, which we take as given, and assume that they follow a stepwise procedure. Each step of reasoning leads to a better understanding of the game, in the form of a richer ‘theory of mind’ of the opponent. We then endogenize players’ depth of reasoning through a cost-benefit analysis. In particular, in Section 2.2 we posit that the number of steps players take (their *cognitive bound*) is a function of their cognitive abilities, which determines the cost of reasoning, and the payoff structure of the game, which determines the benefit. The only key assumption on the value of reasoning is that it increases with the stakes of the game. Our framework therefore accommodates a variety of situations in which players’ strategic sophistication increases with the stakes of the game.

Having discussed how the cost-benefit analysis determines players’ cognitive bound, we endogenize their behavior in Section 2.3. Players’ ‘behavioral level’ depends not only on their

cognitive bounds, but also on their beliefs about the opponent’s cognitive abilities and their higher-order beliefs. To illustrate the interaction between payoffs, beliefs and higher-order beliefs, we first consider a simplified version of our model and use it to derive the predictions that we will test experimentally. We then describe the general model and close the section by briefly discussing the related theoretical literature.

The following game will be used as a leading example throughout this section (different games will be considered in Section 5):

**The (modified) 11-20 game:** Two players simultaneously announce an (integer) number between 11 and 20. Players always receive a number of tokens equal to the number they announce. However, if a player announces a number *exactly one less* than his opponent, then he receives an extra reward of  $x$  tokens, where  $x \geq 20$ . If both players choose the same number, then they both receive an extra 10 tokens. Each token corresponds to one unit of payoff.<sup>5</sup>

## 2.1 Steps of Reasoning

To keep the notation simple, we focus on two-player games with complete information:  $G = (A_i, u_i)_{i=1,2}$  is such that  $A_i$  is the (finite) set of actions of player  $i$  and  $u_i : A_1 \times A_2 \rightarrow \mathbb{R}$  is player  $i$ ’s payoff function. We maintain throughout that  $i \neq j$ . We denote player  $i$ ’s best response correspondence by  $BR_i : \Delta(A_j) \rightrightarrows A_i$ . For simplicity we assume that  $G$  is such that  $BR_i(a_j)$  is a singleton whenever  $a_j$  is a pure action. Then we assume that a player’s reasoning is represented by a sequence of (possibly mixed) action profiles  $\{(a_1^k, a_2^k)\}_{k \in \mathbb{N}}$  such that  $a_i^{k+1} = BR_i(a_j^k)$  for each  $k = 0, 1, \dots$ .<sup>6</sup> We refer to these sequences as *paths of reasoning*, and to profile  $a^0 = (a_1^0, a_2^0)$  as ‘the anchor’. Action  $a_i^0$  is what player  $i$  would play by default, without any strategic understanding of the game. As player  $i$  performs the first step of reasoning, however, he becomes aware that his opponent could play  $a_j^0$ , and thus considers playing  $a_i^1 = BR_i(a_j^0)$ . Similarly, as player  $i$  advances from step  $k - 1$  to step  $k$ , he realizes that his opponent may play  $a_j^{k-1}$ , in which case the best response would be  $a_i^k = BR_i(a_j^{k-1})$ .

We interpret the steps of reasoning as ‘rounds of introspection’. In our model, players are not boundedly rational in the sense of failing to compute best responses. Rather, players are limited in their ability to conceive that the opponent may perform the same steps of reasoning. As an illustration, consider the 11-20 game described above. For any player  $i$ , action  $a_i^0 = 20$  is a natural action for a level-0 player, as it is the number that a player would report if he ignored all strategic considerations.<sup>7</sup> If player  $i$  exerts cognitive effort and performs the first step of the reasoning process, then he realizes that his opponent may play 20, in which case his best response would be 19. If he performs a second step, then he realizes that  $j$  may also

<sup>5</sup>This game is a modified version of Arad and Rubinstein’s (2012) ‘11-20’ game. This is also the game used in the experiment of Section 3. We defer to that section the discussion of the properties of this game, and its suitability to our objectives.

<sup>6</sup>If  $a_j^k$  is a mixed action,  $i$ ’s best response need not be unique. In that case, we assume that the action is drawn from a uniform distribution over the best responses. We abuse notation and write  $a_i^{k+1} = BR_i(a_j^k)$  in both cases. (We maintain the standard notation  $\Delta(A_j)$  to refer to the set of distributions over  $A_j$ .)

<sup>7</sup>As we will discuss in Section 3, different specifications of the level-0 (including the uniform distribution) would not affect the analysis. For simplicity, we only consider  $a_i^0 = 20$  here.

have performed one step of reasoning, and choose 19, in which case his best response would be  $a_i^2 = 18$ , and so on. This process, however, does not necessarily pin down a player’s behavior, which also depends on his beliefs about the opponent’s cognitive abilities. For instance, if player  $i$  has performed three steps of reasoning then he understands enough to play 17 *if* he believes that  $j$  plays according to two steps of reasoning. But if  $i$  thinks that  $j$  has performed fewer steps, then  $i$  would *not* play 17.

This reasoning process therefore consists of developing an increasingly sophisticated ‘theory of mind’: further iterations uncover richer interactive hypotheticals, which describe a player’s understanding of the strategic situation, thereby extending the domain of his possible beliefs about the opponent’s reasoning process. Once formally introduced (Section 2.3), such beliefs will contribute in determining behavior.

## 2.2 The Cognitive Bound: Individual Understanding of the Game

The model we propose for endogenizing the steps of reasoning is based on a cost-benefit analysis. Performing additional rounds of reasoning entails incurring a cognitive cost. While these costs reflect a player’s cognitive ability, which we view as exogenous, we assume that the benefits of performing an extra step of reasoning depend on the payoff structure of the game. This captures the idea that different games may provide different incentives to think.

We stress that we do not view this cost-benefit analysis as an optimization problem actually solved by the agent, but rather as a modeling device to represent a player’s reasoning. We hypothesize that an agent’s understanding of the game varies systematically with the payoff structure, and hence it can be modeled *as if* the cognitive bound  $\hat{k}_i$  results from a cost-benefit analysis. This is formally shown in Alaoui and Penta (2015), where we provide an axiomatic foundation to our approach and derive the cost-benefit representation from primitive assumptions on the player’s reasoning process.

### 2.2.1 Endogenous Cognitive Bound

Formally, we assume that the value of doing extra steps of reasoning only depends on the payoff structure of the game. Fixing the game payoffs, we define function  $v_i : \mathbb{N} \rightarrow \mathbb{R}_+$ , where  $v_i(k)$  represents  $i$ ’s value of doing the  $k$ -th round of reasoning, given the previous  $k - 1$  rounds. The cognitive ability of agent  $i$  is represented by a cost function  $c_i : \mathbb{N} \rightarrow \mathbb{R}_+$ , where  $c_i(k)$  denotes  $i$ ’s incremental cost of performing the  $k$ -th round of reasoning. Since players can be heterogeneous in their cognitive abilities, which is captured by different cost functions, it will be useful to introduce a notion of relative sophistication:

**Definition 1** *Consider two cost functions,  $c'$  and  $c''$ . We say that  $c'$  is ‘more sophisticated’ than  $c''$ , if  $c'(k) \leq c''(k)$  for every  $k$ . For any  $c_i \in \mathbb{R}_+^{\mathbb{N}}$ , we denote by  $C^+(c_i)$  and  $C^-(c_i)$  the sets of cost functions that are respectively ‘more’ and ‘less’ sophisticated than  $c_i$ .*

We introduce next a mapping to identify the intersection between the value of reasoning and the cost function: Let  $\mathcal{K} : \mathbb{R}_+^N \times \mathbb{R}_+^N \rightarrow \mathbb{N}$  be such that, for any  $(c, v) \in \mathbb{R}_+^N \times \mathbb{R}_+^N$ ,

$$\mathcal{K}(c, v) = \min \{k \in \mathbb{N} : c(k) \leq v(k) \text{ and } c(k+1) > v(k+1)\}, \quad (1)$$

with the understanding that  $\mathcal{K}(c, v) = \infty$  if the set in equation (1) is empty. Player  $i$ 's *cognitive bound*, which represents his understanding of the game, is then determined by the value that this function takes at  $(c_i, v_i)$ :

**Definition 2 (Cognitive Bound)** *Given cost and value functions  $(c_i, v_i)$ , the cognitive bound of player  $i$  is defined as:*

$$\hat{k}_i = \mathcal{K}(c_i, v_i). \quad (2)$$

Player  $i$  therefore stops the iterative process as soon as the cost of performing an additional round of introspection exceeds the value. The point at which this occurs identifies his cognitive bound  $\hat{k}_i$ . Note that a player does not compare the benefits and costs at higher  $k$ 's. A player who has performed  $k$  rounds of introspection is only aware of the portion uncovered by the  $k$  steps, and performs a ‘one-step ahead’ comparison of the cost and value of reasoning.<sup>8</sup>

We maintain throughout the following assumptions on the cost and value of reasoning:

**Assumption 1 (Value of Reasoning)** *The value of reasoning only depends on the payoffs of the game, and  $v_i = v_j$  if the game is symmetric. Furthermore,  $v_i(k) \geq 0$  for every  $k \in \mathbb{N}$ .*

**Assumption 2 (Cost of Reasoning)** *For every  $k \in \mathbb{N}$ ,  $c_i(k) \geq 0$  and  $c_i(0) = 0$ .*

Assumption 2 is self-explanatory: it merely defines the notion of cost, naturally zero at the default step. The assumption that  $v_i$  only depends on the game's payoffs implies that an agent's cost-benefit analysis is independent of the opponent's identity, for a given game and a given reasoning procedure. In this respect, the cognitive bound  $\hat{k}_i$  can be seen to be determined separately from the player's beliefs about the opponent, although both factors affect his behavior (Section 2.3). This captures the notion that this process represents the agent's capacity to understand the game, which cannot be affected by the opponent (unlike  $i$ 's choice, which depends on  $i$ 's beliefs about the opponent). This observation also justifies the assumption that  $v_i(k) \geq 0$ : net of its cost, a deeper understanding of the game is never detrimental to the agent, who can at worst ignore the extra insight of each further step of reasoning.<sup>9</sup> By performing the second step of reasoning,  $i$  realizes that  $j$  could play 19, in

<sup>8</sup>The ‘myopic’ (one-step-ahead) procedure that we assume captures the idea, inherent to the very notion of bounded rationality, that the agents do not know (or are not aware of) what they have not yet thought about. Formalizing this notion is often perceived to be a fundamental difficulty in developing a theory of bounded rationality; in this model it emerges naturally.

<sup>9</sup>Note that  $j$  does not observe the rounds of reasoning as  $i$  performs them, and so additional reasoning cannot have negative value from becoming common knowledge. Also,  $v_i$  does not represent the ‘actual’ gain of performing extra steps of reasoning, which is unknown to player  $i$  and which depends on the opponent's behavior. In initial response settings, the value of reasoning need not coincide with the actual gain in payoffs.

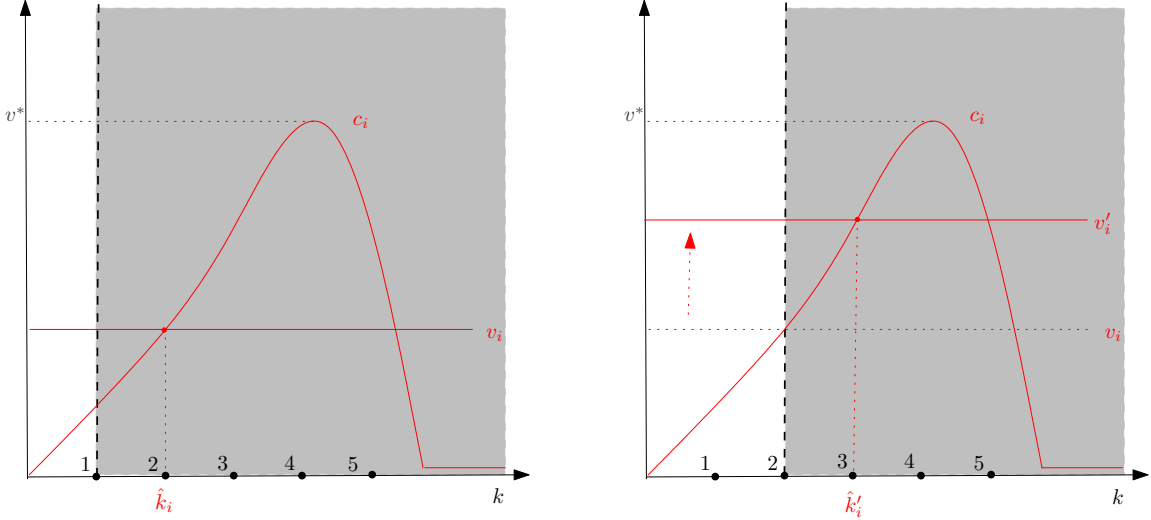


Figure 1: As  $v_i$  increases (from the left figure to the right figure),  $\hat{k}_i$  (weakly) increases. In both figures, the grey area represents the ‘unawareness region’ of player  $i$ .

which case  $i$  would best respond by playing 18. But  $i$  is not forced to play 18; he can play according to less rounds if he believes that  $j$  has not performed the first step of reasoning.<sup>10</sup>

It is easy to check that, under the assumptions above, the cognitive bound  $\hat{k}_i$  is monotonic in players’ sophistication and in the incentives to reason:

**Proposition 1 (Depth of Reasoning)** *Under Assumptions 1 and 2: (i) For any  $c_i$ ,  $v'_i(k) \geq v_i(k)$  for all  $k$  implies  $\mathcal{K}(c_i, v'_i) \geq \mathcal{K}(c_i, v_i)$ ; (ii) For any  $v_i$ ,  $c'_i(k) \geq c_i(k)$  for all  $k$  implies  $\mathcal{K}(c_i, v_i) \geq \mathcal{K}(c'_i, v_i)$ .*

**Example 1** *In Figure 1, the cost function  $c_i$  is non-monotonic and the value of reasoning  $v_i$  is constant, but these shapes are chosen for illustrative purposes only. Player  $i$ ’s cognitive bound,  $\hat{k}_i$ , is determined by the first intersection of  $c_i$  and  $v_i$ , as in Definition 2. In the graph on the left,  $\hat{k}_i = 2$ , meaning that player  $i$  has ‘become aware’ of one round of reasoning of the opponent. The grey area represents player  $i$ ’s ‘unawareness region’ about the opponent’s steps of reasoning. As the value  $v_i$  increases,  $\hat{k}_i$  remains constant at first, but then increases to  $\hat{k}'_i = 3$  when level  $v'_i$  is reached. Correspondingly, the grey area of unawareness shifts to the right, uncovering one more round of reasoning. If  $v_i$  is further increased,  $i$ ’s cognitive bound  $\hat{k}_i$  eventually increases to 4 once  $v^*$  is reached, after which  $\hat{k}_i$  jumps to  $\infty$ .*

*The non-monotonic cost function  $c_i$  thus captures the situation of a player who suddenly understands the game after having performed a few rounds of reasoning. At the other extreme, if  $c_i$  were vertical after any  $k$ , then there would be an absolute cognitive bound, which would not be affected by an increase in  $v_i$ .*

<sup>10</sup>It may appear plausible that the player stops reasoning if he believes that his opponent has already reached his bound, because the extra steps of reasoning would not affect the player’s own choice. This alternative formulation can be easily accommodated in our model, which accounts for beliefs, through a reinterpretation of some of the variables.



**Depth of Reasoning Across Games.** In what follows, we make comparisons of behavior and depth of reasoning across strategic settings. In particular, we analyze how changing the game’s payoffs affects players’ cognitive bound through changes in the value of reasoning. Phrased differently, we conduct a comparative statics exercise on  $\mathcal{K}(c_i, v_i)$  as  $v_i$  is changed. But for this exercise to be meaningful, it is important to shift  $v_i$  without shifting the cost function  $c_i$ . For instance, we would not compare the 11-20 game with low stakes to the normal form of chess with a high reward for winning, and conclude that the higher incentives imply that the depth of reasoning must be higher in chess. This logic would be flawed because the cost of reasoning is arguably higher in chess, hence both the cost and the value of reasoning vary in the same direction, so that the overall effect on the cognitive bound is ambiguous.

We avoid this issue by comparing games that are sufficiently similar from a cognitive viewpoint that they entail the same cognitive cost. For instance, the 11-20 game with  $x = 20$  or  $x = 80$  (the extra payoff for being exactly one below the opponent), have essentially the same structure, and so are equally difficult to understand, even though they may provide different incentives to reason. The following notion of *cognitive equivalence* formalizes the idea:

**Definition 3 (Cognitive Equivalence)** Games  $G = (A_i, u_i)_{i=1,2}$  and  $\hat{G} = (\hat{A}_i, \hat{u}_i)_{i=1,2}$  are cognitively equivalent if, for each  $i \in \{1, 2\}$ ,  $A_i = \hat{A}_i$  and the paths of reasoning associated with each game are identical, i.e.  $\{a^k\}_{k \in \mathbb{N}} = \{\hat{a}^k\}_{k \in \mathbb{N}}$ .

Consistent with the axiomatic foundation in Alaoui and Penta (2015), we assume the following:

**Assumption 3** An individual’s cost of reasoning is the same in cognitively equivalent games.

Thus, differences between cognitively equivalent games will only determine differences in the value of reasoning, if any, thereby allowing meaningful comparative statics. For instance, the cost of reasoning of a particular player would be the same in the 11-20 game, for different values of  $x \geq 20$ , and it would be the same in the chess game, for any reward. The two games, however, are not cognitively equivalent to each other, and therefore may be associated with different costs of reasoning.

We now discuss how the value of reasoning is affected by changing the stakes of the game. Since payoffs are expressed in utils, if two cognitively equivalent games have identical payoff differences in  $i$ ’s actions, then they determine the same choice problem for player  $i$ , hence the same stakes. Varying the stakes for player  $i$ , and consequently his incentives to reason, requires varying the payoff differences for at least some of  $i$ ’s actions. Furthermore, if an agent’s value of reasoning is purely instrumental, then the relevant factor is the possibility of understanding that, given his opponent’s hypothetical behavior, another action may be preferable to his current  $a_i^{k-1}$ . The relevant payoff differences, therefore, are those between actions that he may consider switching to and his current  $a_i^{k-1}$ , unless they do not impact his decision to switch (as would be the case for those associated with an  $a_j$  for which  $a_i^{k-1}$  is already optimal).<sup>11</sup>

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<sup>11</sup>For instance, if all actions are dominated by the current action  $a_i^{k-1}$ , then a small change in the payoffs

The incentives to reason are unambiguously higher if all such payoff differences are higher. Summarizing, if relevant payoff differences are the same then the value of reasoning is the same; if they all (weakly) increase then the value of reasoning (weakly) increases.

**Assumption 4 (Changing Incentives)** *Let  $G = (A_i, u_i)_{i=1,2}$  and  $\hat{G} = (A_i, \hat{u}_i)_{i=1,2}$  be two cognitively equivalent games, with associated value of reasoning  $v_i$  and  $\hat{v}_i$ , respectively. For any  $k$ , if  $\hat{u}_i(a_i, a_j) - \hat{u}_i(a_i^{k-1}, a_j) = u_i(a_i, a_j) - u_i(a_i^{k-1}, a_j)$  (resp.  $\geq$ ) for all  $a_i$  and  $a_j$  s.t.  $a_i^{k-1} \notin BR_i(a_j)$ , then  $\hat{v}_i(k) = v_i(k)$  (resp.  $\hat{v}_i(k) \geq v_i(k)$ ).*

Assumption 4 is very general, and only consists of minimal conditions that would be satisfied whenever the value of reasoning is purely instrumental. In the 11-20 game, this assumption simply implies that the value of reasoning at every step is (weakly) increasing in  $x$ . It imposes no further restrictions.

**Example 2** *An example of a functional form that satisfies Assumptions 1 and 4 is provided by the ‘maximum-gain representation’ that we will consider in Section 5:*

$$v_i(k) = \max_{a_j \in A_j} u_i(BR_i(a_j), a_j) - u_i(a_i^{k-1}, a_j). \quad (3)$$

*In words, the value of reasoning for player  $i$ , at each step, is equal to the maximum difference between the payoff that the player could get if he chose the optimal action  $BR_i(a_j)$  and the payoff he would receive given his current action  $a_i^{k-1}$ , out of all the possible opponent’s actions. Effectively, individuals are optimistic over the gain in thinking more, or, alternatively, cautious about the validity of their current understanding.*

*A more general representation could take the form of an ‘expected gain’:*

$$v_i(k) = \sum_{a_j \in A_j} p(a_j)(u_i(BR_i(a_j), a_j) - u_i(a_i^{k-1}, a_j)). \quad (4)$$

*In this case, it is as if the agent believes that, with probability  $p(a_j)$ , the next step of reasoning will reveal that the opponent chooses  $a_j$ , in which case he would switch from the current action  $a_i^{k-1}$  to the best response to  $a_j$ . The resulting value  $v_i(k)$  is the expected gain averaging over all  $a_j$ , given the weights  $p(\cdot)$ . Both these representations are given axiomatic foundations in Alaoui and Penta (2015).*

## 2.2.2 Discussion of the Maintained Assumptions of the ‘Detail-Free’ Model

Assumptions 1-4 entail minimal restrictions on the cost and value of reasoning functions. In particular, these conditions contain virtually no assumptions about their shape (their monotonicity, convexity, etc.). Maintaining this level of generality allows us to focus on the essential

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associated with one of these actions (e.g.  $a_i$ ), maintaining dominance, would never lead him to switch to  $a_i$  (recall that we are not considering difficulty in computing the best response). In this example, the change in payoffs plays no role in the player’s decision.

features of our approach and to capture different kinds of plausible cost functions. For instance, in the 11-20 game, a player who understands the inductive structure of the problem would have a non-monotonic cost  $c_i$ . His first rounds of reasoning would be cognitively costly but those following the understanding of the recursive pattern would not be (cf. Example 1). We do not assume that the cost function has this shape for the experiment, but we allow it.

Clearly, stronger assumptions such as the ones illustrated in Example 2 would enable sharper predictions. Deriving falsifiable predictions that do not depend on parametric assumptions, however, is key to isolate the conceptual and empirical relevance of the cost-benefit approach in this novel domain. In the next section we show that, once agents’ beliefs are modeled, this minimal set of assumptions enables a rich set of testable (i.e. falsifiable) predictions. Stronger restrictions on the functional forms will be imposed for the calibration exercise of Section 5.

## 2.3 From Reasoning to Behavior

As discussed, the cognitive bound of a player does not necessarily determine his behavior. Given a player’s understanding of the game, his action also depends on his beliefs about the opponent. But if the agent’s choices depend on his beliefs about the opponent, then they may also depend on his beliefs about his opponent’s beliefs about *him*, and so forth. That is, disentangling depth of reasoning from beliefs about opponents requires accounting for higher-order reasoning as well. Reconciling higher-order reasoning with bounded depth of reasoning raises modeling challenges. In our general framework, we model belief hierarchies by means of ‘cognitive type spaces’:

**Definition 4 (Model of Beliefs)** *A ‘cognitive type space’ (CTS) is a tuple  $(T_i, (c_{t_i}, \beta_{t_i})_{t_i \in T_i})_{i=1,2}$  s.t.  $T_i$  is a finite set of types of player  $i$ , and for each type  $t_i$ ,  $c_{t_i} : \mathbb{N} \rightarrow \mathbb{R}_+$  and  $\beta_{t_i} \in \Delta(T_j)$  denote type  $t_i$ ’s cost of reasoning and beliefs about the opponent’s type, respectively.*

This definition accommodates very general hierarchies of beliefs. To understand how beliefs and cognitive bounds jointly determine behavior, it is useful to first consider a simpler class of beliefs, introduced in Section 2.3.1, which also provides the theoretical underpinnings of the experiment. We return to the general model in Section 2.3.2.

### 2.3.1 Simplified Model: Degenerate Beliefs and Second-Order Types

Players’ beliefs about their opponents’ sophistication need not be correct, as we do not seek an equilibrium concept and correctness of beliefs is not guaranteed by introspection alone. Therefore, the natural units of analysis are individuals, and particularly their reasoning processes and beliefs, as represented by types in a cognitive type space. Types should thus be regarded in isolation, player by player and type by type.<sup>12</sup>

The general model allows for complex higher-order beliefs. In this section we focus on a simple class of types, *second-order types with degenerate beliefs*, which are pinned down by

<sup>12</sup>This approach, also known as the *interim approach*, is the standard one to study non-equilibrium concepts with incomplete information (see, e.g., Weinstein and Yildiz (2007, 2013) or Penta (2012, 2013)).

three objects: the cost function,  $c_i$ , the beliefs about the opponent's,  $c_j^i$ , and the beliefs about the opponent's beliefs,  $c_i^{ij}$ . These simple types suffice to illustrate the effects that beliefs and higher-order beliefs have on behavior, and to derive the predictions tested by the experiment in Section 3. We first discuss how these simple types, characterized by the triple  $(c_i, c_j^i, c_i^{ij})$ , are formally represented as types in a CTS.

**Formal Discussion of Second-Order Types.** Formally, a ‘second-order type’ for player  $i$  is any hierarchy of beliefs that can be represented by a model with the following simple structure: player  $i$  can be one of two types,  $T_i = \{c_i, c_i'\}$ , whereas player  $j$  has only one type  $T_j = \{c_j\}$ . Type  $c_j$  attaches probability  $q$  to type  $c_i$ , and  $(1 - q)$  to type  $c_i'$ . In a model with degenerate beliefs,  $q$  can take two values: 0 or 1.

Each type in a CTS provides a full representation of a player's hierarchy of beliefs. Type  $c_i$ , for instance, represents a situation in which player  $i$ 's cost of reasoning is  $c_i$ , and his beliefs about  $j$ , which we denote by  $c_j^i$ , are  $c_j^i = c_j$ . If  $q = 1$ , player  $i$ 's second order beliefs, denoted by  $c_i^{ij}$ , are degenerate and such that  $c_i^{ij} = c_i$ . In this case, type  $c_i$  is a *common-belief type*: it represents the situation in which player  $i$  thinks that both players believe that they both believe, ..., that the costs of reasoning are, respectively,  $c_i$  and  $c_j$ . If  $c_j^i \in C^-(c_i)$ , for instance, player  $i$  believes that his opponent is less sophisticated, and that this is common belief. If instead  $q = 0$ , then  $c_i$ 's second order beliefs are such that  $c_i^{ij} = c_i' \neq c_i$ . That is, this type believes that player  $j$  believes that  $i$ 's cost function is different from what it actually is. With  $q = 0$  therefore  $c_i$  does not represent a common belief situation, and captures player  $i$ 's concern about  $j$ 's beliefs being incorrect. For instance, if  $c_j^i \in C^-(c_i)$  but  $c_i^{ij} \in C^-(c_i')$ , then  $i$  believes that the opponent is less sophisticated, but that he thinks that  $i$  is even less sophisticated.

Since any second-order type with degenerate beliefs is characterized by a triple  $(c_i, c_j^i, c_i^{ij})$ , in the rest of this section we write types directly as  $t_i = (c_i, c_j^i, c_i^{ij})$ . These functions, together with the value functions  $v_i$  and  $v_j$ , determine player  $i$ 's behavior, as we discuss next.

**Incentives, Beliefs and Behavior.** Let  $(v_i, v_j)$  denote the value of reasoning for players  $i$  and  $j$  in a specific game, and let player  $i$ 's type be  $t_i = (c_i, c_j^i, c_i^{ij})$ . Recall that  $\hat{k}_i$  denotes  $i$ 's cognitive bound, which is at the intersection of his cost function  $c_i$  and his value function  $v_i$  (Definition 2). We also define  $\hat{k}_j^i$  and  $\hat{k}_i^{ij}$  to be  $i$ 's beliefs over  $j$ 's cognitive bound and his beliefs over  $j$ 's beliefs over his ( $i$ 's) cognitive bound, respectively. Similarly, we define  $k_i, k_j^i$  and  $k_i^{ij}$  to be, respectively,  $i$ 's behavioral level (or ‘level of play’), his beliefs over  $j$ 's level of play, and his beliefs over  $j$ 's beliefs over  $i$ 's behavioral level. We maintain that players are rational in that they best respond to their beliefs about the opponent's behavior,  $k_j^i$ . Hence,  $i$ 's behavioral level is  $k_i = k_j^i + 1$ .

We first consider common-belief types, i.e. such that  $c_i^{ij} = c_i$ . We distinguish two cases: (i) If  $\hat{k}_i \leq \mathcal{K}(c_j^i, v_j)$ ,  $i$  believes it common knowledge that  $j$  has no reason to play according to a lower level than  $j$  has attained. Player  $i$  therefore believes that  $j$  plays according to the highest level of sophistication  $i$  can ascribe to  $j$ ,  $\hat{k}_j^i$ , which is equal to  $\hat{k}_i - 1$ . We thus have

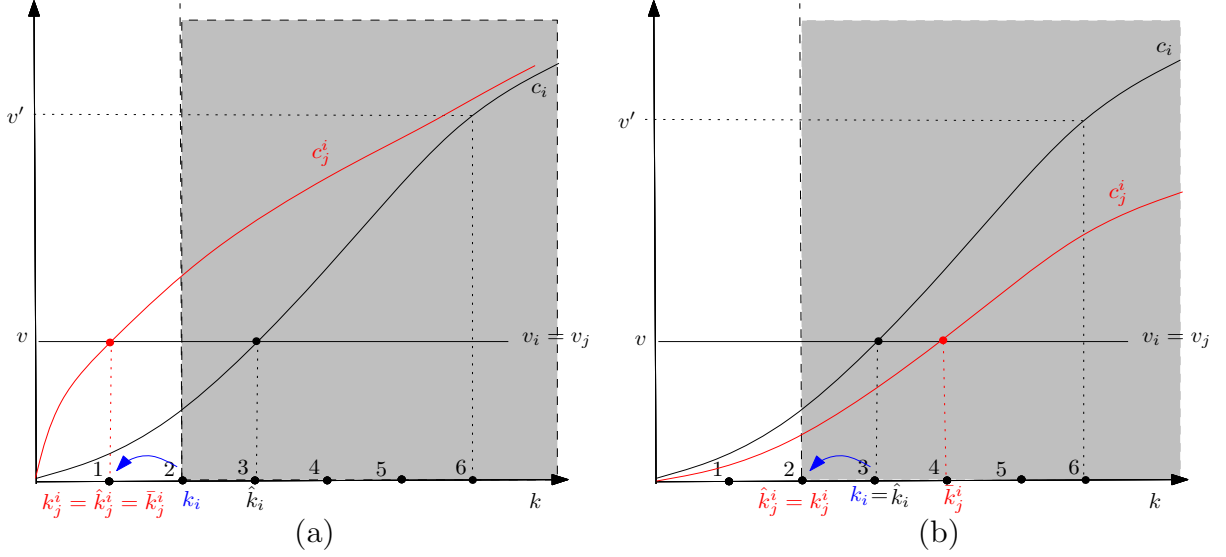


Figure 2: Reasoning about the opponents: on the left,  $c_j^i \in C^-(c_i)$ ; on the right,  $c_j^i \in C^+(c_i)$ . The grey area represents the ‘unawareness region’ of player  $i$ . The intersection of  $c_j^i$  and  $v_j$  is denoted  $\bar{k}_j^i$ .

$k_j^i = \hat{k}_j^i = \hat{k}_i - 1$ , and hence  $i$ ’s behavioral level coincides with his cognitive bound:  $k_i = \hat{k}_i$ .  
(ii) If  $\hat{k}_i > \mathcal{K}(c_j^i, v_j)$ , then  $i$  thinks that  $j$ ’s situation falls within case (i) above. Accordingly, he expects him to play at his cognitive bound, as perceived by  $i$ , that is  $k_j^i = \hat{k}_j^i$ , which in this case is equal to  $\hat{k}_j^i = \mathcal{K}(c_j^i, v_j)$ .

In summary, player  $i$ ’s beliefs about his opponent’s cognitive bound,  $\hat{k}_j^i$ , is at the intersection of cost function  $c_j^i$  and value function  $v_j$  if he is aware of this intersection. But the maximum bound that  $i$  can conceive of for his opponent is constrained to be within the limit of  $i$ ’s own understanding, which is the ‘region of awareness’ up to  $\hat{k}_i - 1$ . Hence, player  $i$ ’s belief about  $j$ ’s bound (hence his beliefs about  $j$ ’s level of play) is

$$\hat{k}_j^i = \min \left\{ \hat{k}_i - 1, \mathcal{K}(c_j^i, v_j) \right\}. \quad (5)$$

It follows that, for a general common-belief type,  $i$ ’s behavioral level  $k_i$  is

$$k_i = k_j^i + 1 = \hat{k}_j^i + 1. \quad (6)$$

Whether  $i$ ’s own cognitive bound  $\hat{k}_i$  constrains his behavioral level  $k_i$  therefore depends on whether  $i$  believes that he has performed more or less rounds of introspection than  $j$ .

**Example 3** In Figure 2.a, player  $i$ , with cost function  $c_i$ , perceives his opponent to be less sophisticated. Since the intersection between  $c_j^i$  and  $v_j$  falls in the region already uncovered by  $i$ ’s cognitive bound,  $i$ ’s belief about  $j$ ’s cognitive bound is at that point, i.e.  $\hat{k}_j^i = 1$ . This also represents  $i$ ’s belief about  $j$ ’s behavior,  $k_j^i$ , hence player  $i$  best responds by playing the action associated with level  $k_i = k_j^i + 1 = 2$ . The cognitive bound  $\hat{k}_i$  is not binding, since  $k_i < \hat{k}_i$ .

Figure 2.b instead represents the same player reasoning about an opponent that he perceives

to be more sophisticated. In this case, the intersection between  $c_j^i$  and  $v_j$  (denoted  $\bar{k}_j^i$  in the graph) falls in the ‘unawareness region’ of player  $i$ . Hence, his perceived cognitive bound for player  $j$  is not  $\bar{k}_j^i$  but  $\hat{k}_j^i = \hat{k}_i - 1$ . Player  $i$  best responds by playing according to level  $k_i = \hat{k}_j^i + 1$ , that is, according to his own cognitive bound  $\hat{k}_i$ .

The next proposition follows from the logic of the example. The common-belief type assumption, which we maintain for simplicity, can be weakened for the results.

**Proposition 2 (Beliefs and Incentives)** *Let  $t_i = (c_i, c_j^i, c_i)$  and  $v_i = v_j$ .*

1. *If  $c_j^i \in C^+(c_i)$ , then the cognitive bound is binding (that is,  $k_i = \hat{k}_i$ ). If only  $v_i$  increases then  $k_i$  and  $\hat{k}_i$  (weakly) increase, and they remain equal if the increase in  $v_i$  is not too large; if only  $v_j$  increases then  $k_i = \hat{k}_i$  does not change; if both  $v_i = v_j$  increase (preserving the symmetry) then  $k_i = \hat{k}_i$  (weakly) increases.*
2. *If  $c_j^i \in C^-(c_i)$ , then  $k_i \leq \hat{k}_i$ . If only  $v_i$  increases then  $\hat{k}_i$  weakly increases but  $k_i$  does not change; if only  $v_j$  increases then  $k_i$  (weakly) increases and  $\hat{k}_i$  remains the same; if both  $v_i = v_j$  increase (preserving the symmetry) then  $k_i$  and  $\hat{k}_i$  (weakly) increase.*

In words, in a game with symmetric value of reasoning, the cognitive bound  $\hat{k}_i$  is always binding for a player who believes that the opponent is more sophisticated. If instead he perceives his opponent to be less sophisticated, then his behavioral  $k_i$  is (weakly) lower than his cognitive bound  $\hat{k}_i$ . This further implies that  $i$  plays according to a (weakly) deeper  $k_i$  when facing a more sophisticated opponent than a less sophisticated one. Moreover, changing the value of reasoning of the opponent, while holding his own constant, changes the player’s behavior only if he believes that his opponent is less sophisticated. Lastly, if the value of reasoning increases for both players, then  $i$ ’s cognitive bound  $\hat{k}_i$  and his behavioral  $k_i$  both (weakly) increase.

**Higher Order Beliefs and Behavior.** Equation (6) derives player  $i$ ’s behavioral level under the assumption that  $t_i$  is a common-belief type (i.e.,  $c_i^{ij} = c_i$ ). Then, Proposition 2 describes the effects of changing incentives and beliefs about the opponents, holding second order beliefs fixed. In general, however, the choice of a player depends on his beliefs about the opponent’s beliefs about *him*. For instance, if player  $i$  is playing an opponent that he regards as less sophisticated, his action may depend on whether or not he believes that the opponent agrees that  $i$  is the relatively more sophisticated player. We therefore consider general second-order types,  $t_i = (c_i, c_j^i, c_i^{ij})$ , without assuming that  $c_i^{ij} = c_i$ , and then study the effects of changing  $i$ ’s second order beliefs,  $c_i^{ij}$ , while holding  $c_j^i$  fixed (Proposition 3).

Formally, while  $i$ ’s beliefs over  $j$ ’s cognitive bound,  $\hat{k}_j^i$ , do not depend on  $c_i^{ij}$ , his beliefs over  $j$ ’s level of play  $k_j^i$  do. In particular,  $k_j^i$  may be less than  $\hat{k}_j^i$  if  $i$  believes that  $j$  underestimates  $i$ ’s sophistication. In other words, player  $i$  puts himself in  $j$ ’s ‘shoes’, to the extent that he can, and perceives  $j$ ’s beliefs over his own cognitive bound,  $\hat{k}_i^{ij}$ , to be:

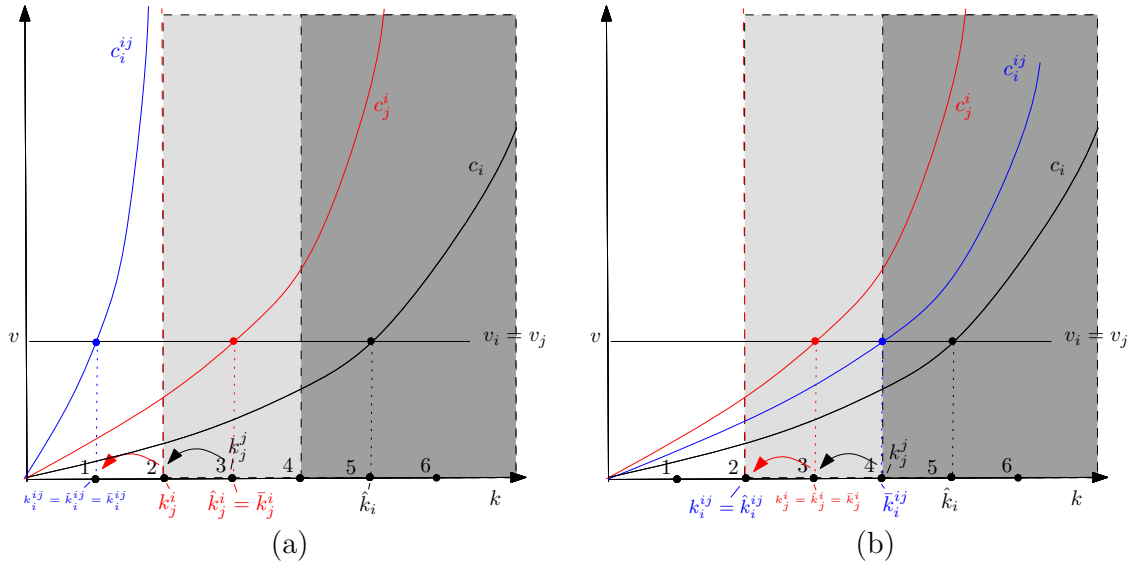


Figure 3: Higher Order Reasoning:  $c_j^i \in C^-(c_i)$ , with  $c_i^{ij} \in C^-(c_j^i)$  on the left, and with  $c_i^{ij} \in C^+(c_j^i)$  on the right. The dark grey area represents the ‘unawareness region’ of player  $i$ , whose cognitive bound is  $\hat{k}_i = 5$ . The light grey area represents the unawareness region of  $j$ , as perceived by  $i$ . The intersection of  $c_j^i$  and  $v_j$  is denoted  $\bar{k}_j^i$ , and the intersection of  $c_i^{ij}$  and  $v_j$  is denoted  $\bar{k}_i^{ij}$ .

$$\hat{k}_i^{ij} = \min \left\{ \mathcal{K}(c_i^{ij}, v_i), \hat{k}_j^i - 1 \right\}. \quad (7)$$

Player  $i$  then expects  $j$  to play according to level  $\hat{k}_i^{ij} + 1$ , provided that he is capable of conceiving of such a level, which is the case if  $\hat{k}_i^{ij} + 1 \leq \hat{k}_i - 1$ . Otherwise, he is limited by his own cognitive bound. Hence, for a general second-order type,  $i$ ’s perception of  $j$ ’s behavioral bound is:

$$k_j^i = \min \left\{ \hat{k}_i^{ij} + 1, \hat{k}_i - 1 \right\}. \quad (8)$$

Player  $i$  then best responds by playing action  $a_i^{k_i}$ , where  $k_i = k_j^i + 1$ .<sup>13</sup>

**Example 4** Figure 3 represents a player with cost function  $c_i$  reasoning about an opponent that he regards as less sophisticated. In Figure 3.a, player  $i$  believes that  $j$  thinks that  $i$  is even less sophisticated (that is,  $c_j^i \in C^-(c_j^i)$ ). Rather than best respond to his perception of  $j$ ’s cognitive bound,  $\hat{k}_j^i$ , player  $i$  best responds to his belief over  $j$ ’s behavioral level,  $k_j^i$ . Here  $k_j^i$  is less than  $\hat{k}_j^i$ , because player  $i$  thinks that  $j$  best responds to his belief that  $i$ ’s bound is at  $k_i^{ij} = 1$ . Hence,  $i$  thinks that  $j$ ’s best response is  $k_j^i = 2$ , and  $i$  in turn best responds with  $k_i = 3$ .

In Figure 3.b,  $c_i^{ij} \in C^+(c_j^i)$ , and therefore  $i$  believes that  $j$  views him as more sophisticated.

<sup>13</sup>Note that setting  $c_i^{ij} = c_i$ , we obtain the case of eq. (6), where  $k_j^i = \hat{k}_j^i$ . This is so because, in that case,  $\mathcal{K}(c_i^{ij}, v_i) = \hat{k}_i$ , hence eq. (7) delivers  $\hat{k}_i^{ij} = \hat{k}_j^i - 1$ . By definition of  $\hat{k}_j^i$ ,  $\hat{k}_j^i - 1 < \hat{k}_i - 1$ , hence eq. (8) implies  $k_j^i = \hat{k}_i^{ij} + 1 = \hat{k}_j^i$ . In fact, as the next example shows, the result from the previous subsection that  $k_j^i = \hat{k}_j^i$  if  $c_j^i \in C^-(c_i)$ , requires only that  $c_i^{ij} \in C^+(c_j^i)$ ; it is not necessary that  $c_i^{ij} = c_i$ .

Player  $i$  therefore expects  $j$  to play at his maximum bound,  $k_j^i = \hat{k}_j^i = 3$ . The best response is thus to play according to  $k_i = 4$ .

This example illustrates that when  $i$  believes that  $j$  is less sophisticated than he is himself, then  $i$ 's choice depends on his second order beliefs:  $i$ 's level of play is lower when he believes that  $j$  is underestimating  $i$ 's sophistication. Extending this example would also show the following result: If  $i$  plays against a more sophisticated opponent and believes that the opponent overestimates him, then  $i$  still plays according to his own bound, as he would if he thought that  $j$  had 'correct' beliefs about him. The higher-order beliefs effects are thus *one-sided*, in the sense that they have behavioral implications only when referring to lower levels of reasoning.

**Proposition 3 (Higher Order Beliefs Effects)** *Let  $t_i = (c_i, c_j^i, c_i^{ij})$  be a second-order type, and let  $v_i = v_j$ .*

1. *Suppose  $c_j^i \in C^-(c_i)$ . For any  $c_i^{ij} \in C^+(c_j^i)$ ,  $k_i = \hat{k}_j^i + 1$ . For  $c_i^{ij} \in C^-(c_j^i)$ ,  $k_i$  (weakly) decreases as  $c_i^{ij}$  becomes less sophisticated.*
2. *Suppose  $c_j^i \in C^+(c_i)$ . For any  $c_i^{ij} \in C^+(c_i)$  (rather than  $c_i^{ij} \in C^+(c_j^i)$ ),  $k_i = \hat{k}_i$ . For  $c_i^{ij} \in C^-(c_i)$ ,  $k_i$  (weakly) decreases as  $c_i^{ij}$  becomes less sophisticated.*

**Testable Predictions for the 11-20 Game.** The following proposition derives the predictions that will be tested in the experiment of Section 3. We emphasize that this proposition follows from Assumptions 1-4 only, which (as discussed in Section 2.2.2) entail minimal restrictions on the functional forms. This proposition therefore allows us to test the detail-free implications of the cost-benefit approach. It also shows that, even in its detail-free specification, our model delivers a rich set of testable predictions and provides a clear framework for the experimental design.

**Proposition 4** *Consider the 11-20 game introduced above. Under Assumptions 1-4, for any  $i$  whose hierarchy of beliefs are described by second-order types, the following holds:*

1. **Changing Incentives:** *For any  $c_i, c_j^i, c_i^{ij}$ , the number chosen by player  $i$  is (weakly) decreasing in  $x$ .*
2. **Changing Beliefs:** *For any  $x$  and for any  $c_i$  and  $c_i^{ij}$ , the number chosen by player  $i$  (weakly) decreases as  $c_j^i$  becomes more sophisticated. Moreover, if  $c_i^{ij} = c_i$ , then  $i$ 's cognitive bound is binding if he regards his opponent as more sophisticated (that is,  $k_i = \hat{k}_i$  if  $c_j^i \in C^+(c_i)$ ), not necessarily otherwise.*
3. **Higher Order Beliefs matter, but their effects are one-sided:** *For any  $x$  and for any  $c_i$  and  $c_j^i$ , the number chosen by player  $i$  (weakly) decreases as  $c_i^{ij}$  becomes more sophisticated. Moreover:*



- (a) If  $i$  regards the opponent as less sophisticated ( $c_j^i \in C^-(c_i)$ ), for any  $x$ , the number chosen by player  $i$  is constant in  $c_i^{ij}$  as long as  $c_i^{ij} \in C^+(c_j^i)$ . For  $c_i^{ij} \in C^-(c_j^i)$ , the number decreases as  $c_i^{ij}$  gets more sophisticated.
- (b) If  $i$  regards the opponent as more sophisticated ( $c_j^i \in C^+(c_i)$ ), for any  $x$ , the number chosen by player  $i$  instead is constant in  $c_i^{ij}$  as long as  $c_i^{ij} \in C^+(c_i)$  (rather than  $c_i^{ij} \in C^+(c_j^i)$ ). For  $c_i^{ij} \in C^-(c_i)$ , the number increases as  $c_i^{ij}$  gets less sophisticated.

### 2.3.2 General Model

In the simplified model of Section 2.3.1, players' first and second order beliefs suffice to pin down the entire hierarchy of beliefs. In general, belief hierarchies can be more complicated, and lead to more complex patterns of behavior. The intuition, however, is the same: given the payoffs of the game and the associated value of reasoning  $v_i$ , each type  $t_i \in T_i$  in a CTS (Def. 4) induces a depth of reasoning  $\hat{k}_{t_i} = \mathcal{K}(c_{t_i}, v_i)$ . Then, the type with the lowest cognitive bound in the CTS (denote it by  $\hat{t}^0$ ) believes it common certainty that no type is less sophisticated. His cognitive bound would thus be binding, and his action would coincide with the corresponding step in the path of reasoning,  $a_i^{\hat{k}_{\hat{t}^0}}$ .

Ignoring for now the case of non-degenerate beliefs, there are two cases for the next deeper type: a type who believes that his opponent is of type  $\hat{t}^0$ , denoted  $t^1$ , best responds accordingly, independent of his own cognitive bound; a type who believes it common certainty that *he* is the least sophisticated type, denoted  $\hat{t}^1$ , plays according to his own bound. The same logic applies to higher types. Hence, for the next deeper type, if a player believes that his opponent is of type  $\hat{t}^0$ ,  $t^1$  or  $\hat{t}^1$ , then he best responds accordingly for each, otherwise he plays according to his own cognitive bound. Clearly, this logic can be iterated, and behavior for general higher-order effects can be derived recursively.

To extend the logic above to non-degenerate beliefs, it suffices to adjust the recursion by requiring that types best respond to the induced distribution of the opponents' actions. Formally, fix a general CTS. For each  $i$ , define  $\alpha_i^0 : T_i \rightarrow A_i$  such that  $\alpha_i^0(\cdot) = a_i^0$ . Recursively, for each  $i = 1, 2$ , and  $k = 1, 2, \dots$ , let  $\alpha_i^k : T_i \rightarrow A_i$  be such that, for each  $t_i \in T_i$ :

$$\alpha_i^k(t_i) = \begin{cases} BR_i \left( \sum_{t_j \in T_j} \beta_{t_i}(t_j) \cdot \alpha_j^{k-1}(t_j) \right) & \text{if } k \leq \mathcal{K}(c_{t_i}, v_i) \\ \alpha_i^{k-1}(t_i) & \text{otherwise.} \end{cases} \quad (9)$$

Note that this recursion coincides with the path of reasoning for  $k < \min_{j \in N, t_j \in T_j} \mathcal{K}(c_{t_j}, v_j)$ , that is, as long as no type has reached his cognitive bound. The recursion becomes constant for a type at iterations above its cognitive bound. This represents the idea that the type's reasoning has stopped. Recursively, the iteration also becomes constant for types that place sufficiently high probability on types whose own recursion has become constant. As for the case with degenerate beliefs, this could be either because they have reached their bound, or (recursively) because they believe the opponent has, and so forth. Thus, a type's optimal behavior (given

his beliefs and cognitive bound) can be obtained once the corresponding recursion has become constant, which happens no later than at the iteration corresponding to that type’s cognitive bound (and exactly at that step, if the bound is indeed binding). Therefore, for each type  $t_i$  in a general CTS, his best response to his beliefs can be defined as

$$\hat{a}_i(t_i) := \alpha_i^{\mathcal{K}(c_{t_i}, v_i)}(t_i). \quad (10)$$

Hence, types with the lowest depth of reasoning play according to their own bound. Deeper types best respond to their beliefs, to the extent that they understand them (given their cognitive bound).<sup>14</sup> Also note that the logic of the recursion captures the idea that higher-order beliefs effects are bounded by each type’s depth of reasoning, which is consistent with the ‘one-sidedness’ results of Propositions 3 and 4. Thus, despite entailing standard hierarchies over cost functions, CTSs combined with recursion (9) are an effective device for modeling the limited effects of higher-order beliefs in the presence of bounded depth of reasoning.

## 2.4 Related Models

Within the literature on level- $k$  reasoning, the closest model to ours is Strzalecki’s (2014), which also separates depth of reasoning from beliefs and behavior. In his model, the depth of reasoning is exogenous, and each type’s beliefs are concentrated on types with lower depth of reasoning. Strzalecki’s model can be nested in ours, letting cost functions be zero and then infinite at some fixed  $k$ , and letting beliefs be concentrated on less sophisticated types (in the sense of Definition 1). In that case, eq. (10) has the same behavioral implications as Strzalecki’s equilibrium. If such beliefs are further assumed to be equal to the correct distribution of types with lower depth of reasoning, then the CH model of Camerer et al. (2004) obtains as a further special case. The separation of beliefs from behavior thus enables us to accommodate in a unified framework both the CH model and models in which each level- $k$  best respond to  $(k - 1)$  (e.g., Nagel (1995), Costa-Gomes and Crawford (2006), Crawford and Iriberri (2007)).

The general notion that players follow a cost-benefit analysis is present in the language of Camerer et al. (2004), but not in their model itself, as players’ cognitive types remain exogenous. A recent paper by Choi (2012) extends Camerer et al.’s (2004) model by letting cognitive types result from an optimal choice, motivated by an evolutionary argument. The objectives and modeling choices are therefore distant. Gabaix (2012) also pursues a cost-benefit approach to develop an equilibrium concept that allows players to have both incorrect beliefs as well as to respond non-optimally. The ‘noisy introspection’ model of Goeree and Holt (2004) extends the level- $k$  approach introducing non-optimal responses in a non-equilibrium model. Introducing noisy responses in our model of endogenous depth of reasoning is an interesting direction for future research.

From a broader perspective, our approach can be cast within the research agenda on ratio-

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<sup>14</sup>It is easy to verify that the simplified model of the previous section obtains as the special case of second-order types with degenerate beliefs: for those types, eq.(10) is equal to  $a_i^{k_j^i+1}$ , for  $k_j^i$  defined in eq. (8).

nal inattention, which also endogenizes individuals’ limited understanding of the environment through a cost-benefit approach.<sup>15</sup> This literature has thus far focused on non-strategic problems. Strategic settings raise specific complications, particularly due to the interaction between individuals’ understanding, their beliefs and their higher-order beliefs. From a conceptual viewpoint, the literature on unawareness is also related (for a thorough survey, see Schipper (2015)). While models of unawareness in strategic settings have a different focus, our framework can be viewed as endogenizing the awareness of the opponents’ best responses.

### 3 Experimental Design

The experiment tests the key implications of the detail-free model, which concern how behavior is affected by the incentives to reason, the beliefs about the opponents and the higher-order beliefs. The experimental design matches closely the theoretical setting, in which each change occurs in isolation. The precise mapping from the theoretical predictions, stated in Proposition 4, and the treatments of the experiment are summarized in Section 3.3. Throughout the treatments, the baseline game remains the 11-20 game discussed in Section 2, with  $x = 20$ :

The subjects are matched in pairs. Each subject enters an (integer) number between 11 and 20, and always receives that amount in tokens. If he chooses *exactly one less* than his opponent, then he receives an extra  $x = 20$  tokens. If they both choose the same number, then they both receive an extra 10 tokens.

This game is a variation of Arad and Rubinstein’s (2012) ‘11-20’ game, the distinction being that the original version does not include the extra reward in case of a tie. As in the original 11-20 game, the best response to 20 (or to the uniform distribution) is 19, the best response to 19 is 18, and so forth. But with the extra reward in case of tie, the best response to 11 is 11, and not 20, as is the case in the original 11-20 game. Thus, our modification breaks the cycle in the chain of best responses. We discuss the reasons for using this game in Section 3.4.

The subjects of the experiment were 120 undergraduate students from different departments at the Universitat Pompeu Fabra (UPF), in Barcelona. Each subject played twice every treatment described in Sections 3.1 and 3.2, and summarized in Table 1. We provide the exact sequences of treatments used in Appendix A.2.

Each subject was anonymously paired with a new opponent after every iteration of the game. To focus on initial responses and to avoid learning from taking place, the subjects only observed their earnings at the end of the session. Moreover, subjects were paid randomly, and therefore did not have any mechanism for hedging against risk by changing their actions.<sup>16</sup> As

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<sup>15</sup>The classical reference for rational inattention is Sims (2003), which spurred a large literature. A recent paper closely related to our work is Caplin and Dean (2013).

<sup>16</sup>These methods are standard in the literature on ‘initial responses’, where the classical equilibrium approach is hard to justify. See, for instance, Stahl and Wilson (1994, 1995), Costa-Gomes, Crawford and Broseta (2001) and Costa-Gomes and Crawford (2006). For an experimental study of equilibrium in a related game, see Capra, Goeree, Gomez and Holt (1999).

Treatment	Own label	Opponent’s label	Own payoffs	Opponent’s payoffs	Replacement of opponent’s opponent
Homogeneous [A]	$I$ ( $II$ )	$I$ ( $II$ )	Low	Low	No
Heterogeneous [B]	$I$ ( $II$ )	$II$ ( $I$ )	Low	Low	No
Replacement [C]	$I$ ( $II$ )	$II$ ( $I$ )	Low	Low	Yes
Homogeneous-high [A+]	$I$ ( $II$ )	$I$ ( $II$ )	High	High	No
Heterogeneous-high [B+]	$I$ ( $II$ )	$II$ ( $I$ )	High	High	No
Replacement-high [C+]	$I$ ( $II$ )	$II$ ( $I$ )	High	High	Yes

Table 1: Treatment summary: Label  $I$  refers to ‘math and sciences’ or to ‘high’ subjects, and label  $II$  refers to ‘humanities’ or to ‘low’ subjects. There are 120 subjects for each treatment (60 subjects for each classification).

an additional control for order effects, the order of treatments was randomized. Furthermore, since subjects played the same treatments twice during a session, we can compare play for each treatment through equality of distribution tests. The details on the pool of subjects, the earnings and the logistics of the experiment are in Appendix A.

### 3.1 Changing beliefs about the opponents

We consider two different classifications of subjects, an *exogenous classification* and an *endogenous classification*, each with 3 sessions of 20 subjects. In the exogenous classification, subjects are distinguished by their degree of study. Specifically, in each session of the experiment, 10 students are drawn from the field of humanities (humanities, human resources, and translation), and 10 from math and sciences (math, computer science, electrical engineering, biology and economics). The subjects are aware of their own classification and are labeled as ‘humanities’ or ‘math and sciences’. In the endogenous classification, there is no restriction on the pool of subjects. Moreover, the subjects are not informed about the field of study of the other players. Before playing the game, however, they take a test of our design. Based on their performance on this test, each student is either labeled as ‘high’ or ‘low’, and is shown his own label before playing the game. We defer the description of this test and a discussion of the rationale for choosing these classifications to Section 3.4.1.

These classifications allow us to change subjects’ beliefs about their opponents. In each treatment, the subjects are given information concerning their opponents. They play the baseline game against someone from their own label (homogeneous treatment [A]) and against someone from the other label (heterogeneous treatment [B]).

To test for higher-order beliefs effects, and whether subjects believe that the behavior of their opponents also changes when they face opponents of different levels of sophistication, we administer replacement treatment [C]. In this treatment, we vary the subject’s belief over his opponent’s opponent. A ‘math and sciences’ subject, for instance, is given the following instructions: “[...] two students from humanities play against each other. You play against the number that one of them has picked.” The reasons for using this exact wording are discussed in Section 3.4.2.

### 3.2 Changing incentives

We next consider a second dimension that would entail a change in players' chosen actions, according to our framework. In particular, we aim to test the central premise of our theoretical model, that players may perform more rounds of introspection if they are given more incentives to do so. To do this, we change the extra gain for choosing the action precisely one below the opponent's from  $x = 20$  to 80. The rest of the game remains the same. It is immediate that this change does not affect the path of reasoning, irrespective of whether the level-0 is specified as 20 or as the uniform distribution. It only increases the rewards for players who stop at the 'correct' round of reasoning. In the context of our theoretical model, this game is in the same cognitive equivalence class as the baseline game, and so the costs of reasoning are identical.

We consider three treatments for this 'high payoff game': homogeneous treatment [A+], heterogeneous treatment [B+], and replacement treatment [C+]. These treatments are the same as treatments [A], [B] and [C], respectively, but with higher payoffs. We then compare an agent's play under different payoffs by comparing [A] to [A+], [B] to [B+] and [C] to [C+]. We also compare treatments [A+], [B+] and [C+] in an analogous way to the comparison between treatments [A], [B] and [C].

This concludes our discussion of the main treatments. The next Section explains how these treatments relate to the theoretical model, and presents the theoretical predictions for the experiment. These predictions are summarized in Table 2.

### 3.3 Theoretical Predictions for the Experiment

Recall that we use the terminology 'label  $I$ ' (resp., 'label  $II$ ') to refer to the 'high score' ('low score') subjects in the endogenous classification or to the 'math and sciences' ('humanities') subjects for the exogenous. Accordingly, we introduce notation  $l_i = \{I, II\}$  to refer to individual  $i$ 's label.

For simplicity, we only consider the second-order types discussed in Section 2.3.1. We assume that an individual's cost of reasoning,  $c_i$ , remains constant throughout all treatments. This is consistent with the cognitive equivalence of the games used in the low and high payoff treatments. We also assume that  $i$ 's first-order beliefs,  $c_j^i$ , only depend on the label of the opponent, and that his second order beliefs  $c_i^{ij}$  only depend on the label of the opponent's opponent (which is  $i$ 's own label, except in the 'replacement' treatments, [C] and [C+]). This implies that an individual is identified by his label  $l_i$ , his cost  $c_i$  and first and second order beliefs in treatment [X], denoted by beliefs  $c_j^{i,[X]}$  and  $c_i^{ij,[X]}$  (for  $X = A, B, C$ ), which satisfy the following:

**E.1:** For all  $i$ :  $c_j^{i,[B]} = c_j^{i,[C]}$ ,  $c_i^{ij,[A]} = c_i^{ij,[B]}$  and for all  $X = A, B, C$ ,  $c_j^{i,[X]} = c_j^{i,[X+]}$  and  $c_i^{ij,[X]} = c_i^{ij,[X+]}$ .

We also assume that individuals commonly believe that label  $I$  players are more sophisticated than label  $II$ . Formally:

Labels	Changing beliefs (low payoffs)	Changing beliefs (high payoffs)	Changing payoffs
$l_i = I$	$F_C \succsim F_B \succsim F_A$	$F_{C+} \succsim F_{B+} \succsim F_{A+}$	$F_X \succsim F_{X+}$ for $X = A, B, C$
$l_i = II$	$F_A \succsim F_B; F_B \approx F_C$	$F_{A+} \succsim F_{B+}; F_{B+} \approx F_{C+}$	$F_X \succsim F_{X+}$ for $X = A, B, C$

Table 2: Summary of the theoretical predictions of the First Order Stochastic Dominance relations between the distribution of actions in different treatments.

**E.2:** For label  $I$  individuals: if  $l_i = I$ ,  $c_j^{i,[B]} \in C^-(c_j^{i,[A]})$ ,  $c_i^{ij,[C]} \in C^-(c_i^{ij,[B]})$ ; For label  $II$  individuals: if  $l_i = II$ ,  $c_j^{i,[A]} \in C^-(c_j^{i,[B]})$ ,  $c_i^{ij,[B]} \in C^-(c_i^{ij,[C]})$ .

Finally, we assume that label  $II$  individuals always regard label  $I$ 's as more sophisticated than they are:

**E.3:**  $c_j^{i,[B]} \in C^+(c_i)$  whenever  $l_i = II$ .

Under E.1 and E.2, for any  $X = A, B, C$ , the only change between treatment  $[X]$  and  $[X+]$  is in the payoffs  $x$ . These comparisons therefore allow us to test the implications of part 1 of Proposition 4. Treatments  $[A]$  and  $[B]$  (or  $[A+]$  and  $[B+]$ ) instead only differ in  $i$ 's first order beliefs, they thus serve to test part 2 of Proposition 4. Finally, treatments  $[B]$  and  $[C]$  (or  $[B+]$  and  $[C+]$ ) only differ in  $i$ 's second order beliefs. Their comparison therefore addresses the third part of Proposition 4.

More specifically, let  $F_X^l$  denote the cumulative distribution of actions  $a \in \{11, \dots, 20\}$  in treatment  $X$  for label  $l \in \{I, II\}$ , and denote by  $\succsim$  the first order stochastic dominance relation.<sup>17</sup> Proposition 4 immediately implies the following results, summarized in Table 2:<sup>18</sup>

**Proposition 5** *For any distribution over individuals that satisfy the restrictions in E.1-3, under the maintained assumptions of the detail-free model (Section 2), the following holds: (i) For any  $X = A, B, C$  and  $l = I, II$ ,  $F_X^l \succsim F_{X+}^l$ ; (ii)  $F_C^I \succsim F_B^I \succsim F_A^I$ ; (iii)  $F_A^{II} \succsim F_B^{II} \approx F_C^{II}$ .*

### 3.4 Experimental Design: Discussion

#### 3.4.1 Designing the Group Classification: Demarcation and Focality.

In order to vary subjects' beliefs about the opponents, we divide the pool of subjects into two labeled groups. We then change subjects' beliefs about the opponents by changing the opponent's group in the different treatments. To effectively implement the theoretical proposition that we test, and in particular conditions E.2-3 in Section 3.3, these labels must satisfy two properties. The first is *demarcation*: the groups in the classifications must be perceived as being sufficiently distinct from each other in terms of their strategic sophistication. The second

<sup>17</sup>Given two cumulative distributions  $F(x)$  and  $G(x)$ , we say that  $F$  (weakly) first order stochastically dominates  $G$ , written  $F \succsim G$ , if  $F(x) \leq G(x)$  for every  $x$ .

<sup>18</sup>Our theoretical predictions on the shift of the distribution do not depend on assumptions of degenerate beliefs; as discussed in Section 2, our model allows for non-degenerate beliefs. Noise in the path of reasoning, in the spirit of Goeree and Holt (2004), can be introduced as well.

is *focality*: since subjects’ behavior depends not only on their beliefs but also on their beliefs about their opponents’ beliefs, it is important that the two groups share sufficient agreement about the way they differ. Note that such beliefs need not be correct; it suffices that the subjects ‘commonly agree’ over their relative sophistication. The two classifications we consider have been chosen to guarantee that these properties hold.

The exogenous classification exploits the intuitive, albeit vague, view that ‘math and sciences’ students are regarded as more accustomed to numerical reasoning than ‘humanities’ students. Furthermore, the specific degrees of study used to populate the ‘math and sciences’ group are commonly viewed as being the most selective degrees at UPF, and require the highest entry marks.<sup>19</sup> We therefore expect the subjects to believe the ‘math and sciences’ group to be comparatively more sophisticated in game theoretical reasoning than the ‘humanities’ group. However, the subjects are not primed into shaping specific beliefs about either particular group.

In the endogenous classification, students are classified solely based on their performance in a test of our design. The goal of the test is twofold. It sorts subjects into two groups, and, by labeling the scores obtained by subjects as ‘high’ or ‘low’, the test itself forms the agents’ beliefs over the content of these labels. The main objective of the test is to convince subjects that the result is informative about their opponents’ game theoretical sophistication. To do so, we ensure that our questions appear difficult to solve, and that subjects would be likely to infer that an individual of higher sophistication would respond better to the questions. Subjects with a score above the median are labeled ‘high’, and the others are labeled ‘low’. They do not see their numerical grade, but they are told whether they are labeled ‘high’ or ‘low’. Details of the test are contained in Appendix D.

### 3.4.2 Testing for Effects of Higher Order Beliefs

The objective of treatment [C] is to test the theoretical predictions on higher-order beliefs effects. The precise wording of treatment [C] is designed to pin down the entire hierarchy of beliefs, as described in Section 2.3.1. For instance, the full description that a math and sciences student is given concerning his opponent in treatment [C] is: “[...] two students from humanities play against each other. You play against the number that one of them has picked.” It is therefore clear that he is playing a humanities playing a humanities subject, who himself is playing a humanities subject, and so forth.

### 3.4.3 Choice of the Baseline Game

As argued by Arad and Rubinstein (2012), the 11-20 game presents a number of advantages in the study of sequential reasoning, which are inherited by our modified version. We recall here the most relevant to our purposes. First, using sequential reasoning is natural, as there are no

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<sup>19</sup>These views emerged from informal conversations with students. They are confirmed by the admission scores, used to select the students admitted in the various fields. These scores can be found at: <http://www.elpais.com/especial/universidades/titulaciones/universidad/universidad-pompeu-fabra/45/nota-corte/>.

other obvious focal ways of approaching the game. Note that our aim is not to establish the type of reasoning process itself, which we take as given, and has been an important contribution of the literature (see, in particular, the seminal papers by Nagel (1995), Camerer et al. (2004) and Costa-Gomes and Crawford (2006)).<sup>20</sup> Secondly, the specification of the anchor is intuitively appealing and unambiguous, since choosing 20 is natural for an iterative reasoning process. Moreover, it is the unique best choice for a player who ignores all strategic considerations. Thirdly, there is robustness to the anchor specification, in that the choice of 19 would be the best response for a wide range of anchors, including the uniform distribution over the possible actions. Lastly, best-responding to any action is simple. Since we do not aim to capture cognitive limitations due to computational complexity, having a simple set of best responses is preferable. In addition to these points, our modification of the 11-20 game breaks the cycle in the chain of best responses, which is crucial for our testable predictions.

## 4 Experimental Results

We present, for brevity, only the experimental results for the grouped exogenous and endogenous classifications.<sup>21</sup> We pool the label *I* subjects ('math and sciences' for exogenous treatments and 'high' for endogenous treatments), and we pool the label *II* subjects ('humanities' for exogenous treatments and 'low' for endogenous treatments). Moreover, we present the results by pooling together the treatments when they are repeated. For these repetitions, our pooling is justified by tests for equality of distribution. We analyze first the results when subjects' payoffs are changed, followed by the results when their beliefs over opponents are varied. We discuss in this section the Wilcoxon signed-rank tests and the regressions, and defer further details to Appendix B. In the (random-effects) ordinary least squares estimations (OLS) that follow, we regress, for each label, the outcome on a dummy for the treatments, and another for the classification (endogenous or exogenous). The latter is never significant.

All regressions and statistical tests are in Appendix B. The OLS regressions are in Table 5 of Appendix B and the Wilcoxon signed-rank tests for changes in payoffs and beliefs over opponents are in Table 6 and Table 7, respectively.

### 4.1 Changing Incentives

As the value of reasoning increases for players and their opponents, the model predicts that they would choose actions associated with higher  $k$ 's. Specifically, comparing treatments across different marginal values of payoffs,  $F_A \succeq F_{A+}$ ,  $F_B \succeq F_{B+}$  and  $F_C \succeq F_{C+}$ . These implications

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<sup>20</sup>For studies that focus more directly on the cognitive process itself, see Agranov, Caplin and Tergiman (2015), and the recent works by Bhatt and Camerer (2005), Coricelli and Nagel (2009), and Bhatt, Lohrenz, Camerer and Montague (2010), which use fMRI methods and find further support for level- $k$  models. See also Georganas, Healy and Weber (2015), Gill and Prowse (2015) and Fehr and Huck (2015) for analyses of cognitive ability in strategic settings. For a thorough description of different thought processes, see Bosch-Domènech, García-Montalvo, Nagel and Satorra (2002). Fragiadakis, Knoepfle and Niederle (2013) show that level- $k$  reasoning processes are deliberate.

<sup>21</sup>The figures for the separate classifications are consistent with the results for the grouped classifications.



hold for both label *I* and label *II* subjects. Beginning with label *I*, it is clear from Figure 4 that the empirical distributions [A], [B] and [C] clearly stochastically dominate, respectively, distributions [A+], [B+] and [C+] everywhere.

These results are therefore consistent with our theoretical predictions. Conducting an OLS regression, we find that the coefficients are highly significant ( $< 1\%$ ) for distributions [A] compared to [A+], [B] to [B+] and [C] to [C+], and of the correct sign. The Wilcoxon signed-rank statistic is highly significant ( $< 1\%$ ) for all of these comparisons of distribution as well. Similar results hold for label *II*, with the only difference that the OLS regressions are significant ( $< 5\%$ ) for the comparisons between [A] and [A+].

These findings are consistent with the theory, and with the view that agents perform more rounds of reasoning if the incentives are increased. These results also indicate that changing from an extra 20 tokens to an extra 80 tokens determines a large enough shift in the value function that it leads agents to increase their level of reasoning. The graphs in Figure 4 depict the shifts in the distributions.

## 4.2 Changing beliefs about the opponents

Consider the comparison between homogeneous treatment [A], heterogeneous treatment [B] and replacement treatment [C]. According to the theoretical model,  $F_C \succ F_B \succ F_A$  for label *I* players. These predictions are consistent with the data displayed in Figure 5. Distribution [C] clearly stochastically dominates [B] everywhere, and [B] stochastically dominates [A] nearly everywhere.<sup>22</sup> We also note that [C] clearly stochastically dominates [A] everywhere.

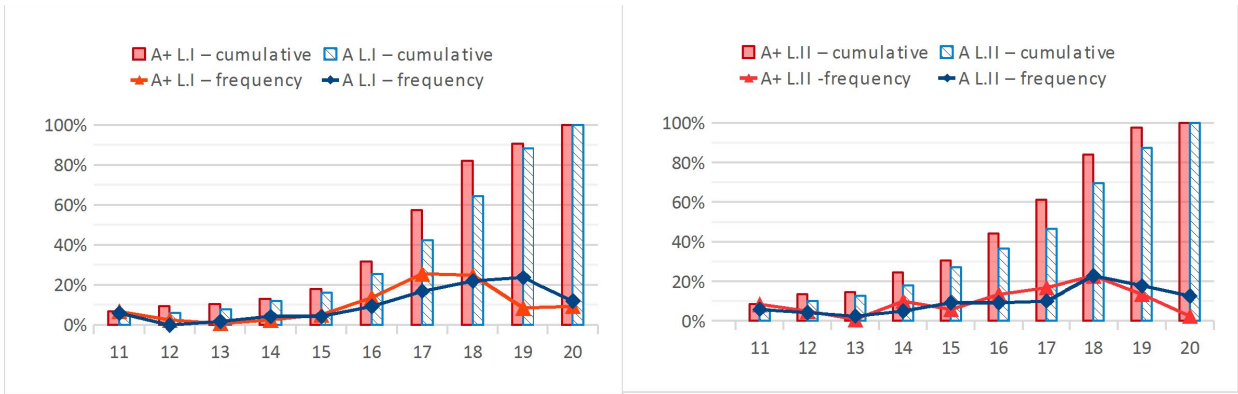
The OLS estimates comparing [A] to [B] are significant ( $< 10\%$ ) and the estimates comparing [A] to [C] are highly significant ( $< 1\%$ ). The estimates comparing [B] to [C], however, are not significant. Figure 5 reveals that distributions [B] and [C] remain very close to each other, and so the lack of significance is not surprising.

Turning next to label *II* players, the model predicts  $F_A \succ F_B \approx F_C$ . Here, no clear difference emerges from Figure 5 between the three cumulative distributions. Conducting Wilcoxon-signed-rank equality of distribution tests confirms the visual intuition, and the OLS estimates are not significant for any of the comparisons of [A] to [B], [B] to [C] or [A] to [C]. While  $F_B \approx F_C$  is the exact prediction of the theoretical model, the result that  $F_A \approx F_B$  indicates that label *II* subjects do not view the sophistication of other label *II* subjects as significantly lower than their own, and therefore do not adjust their level of play in a measurable way. Additional observations are discussed in Appendix E.

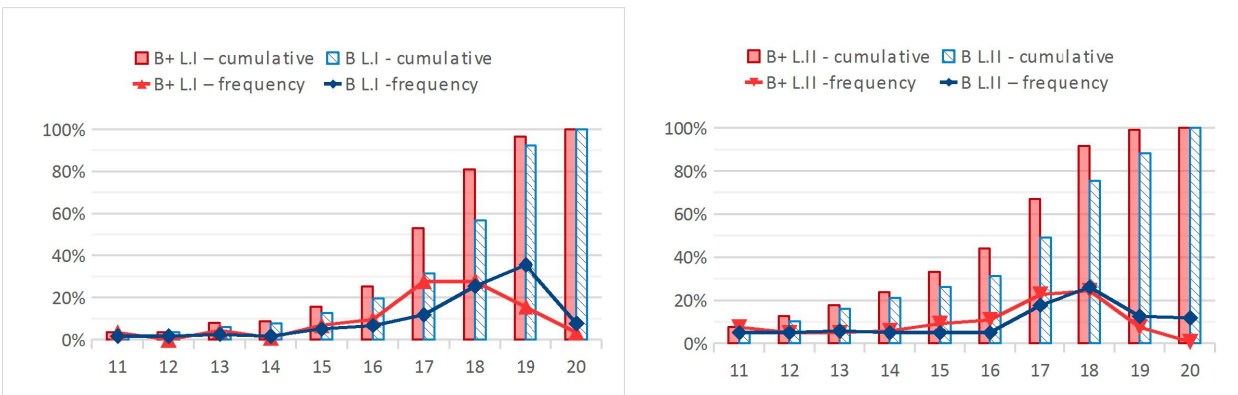
In summary, the experimental results are consistent with our model's predictions. More broadly, our findings also show that individuals change their actions as their incentives and beliefs about the opponents are varied, and that they do so in a systematic way. This illustrates the empirical need for a model that endogenizes depth of reasoning, and supports our approach.

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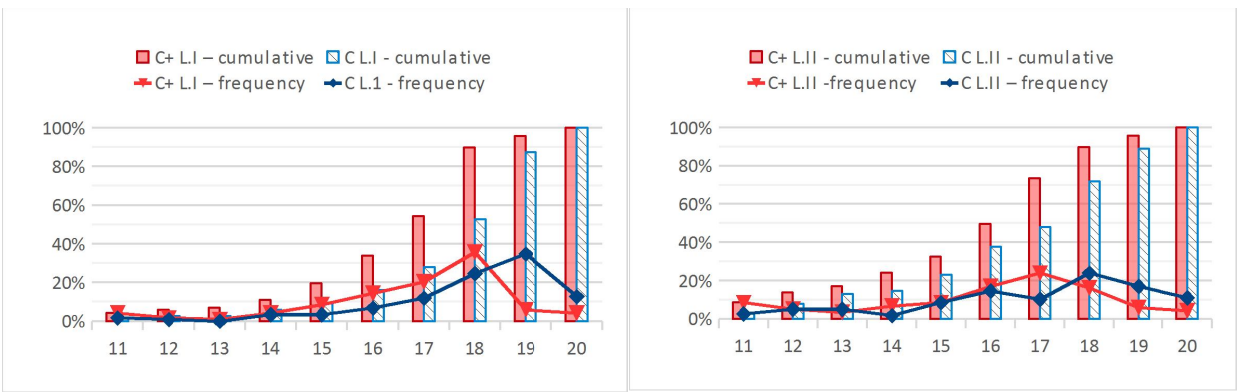
<sup>22</sup>The only exception is at action 19, which is consistent with the well-known observation that stochastic dominance relations are often violated near the endpoints, even when the true distributions are ranked.



(a) : A and A+



(b) : B and B+



(c) : C and C+

Figure 4: **Changing Payoffs, label *I* (left) and label *II* (right).** (Recall that label *I* denotes the high score and math and sciences combined, and label *II* the low scores and humanities combined. Also, for  $X=A,B,C$ ,  $[X+]$  denotes treatment  $[X]$  with high payoffs.)  
**Summary:** For both labels, increasing incentives shifts the level of play towards more sophisticated behavior (i.e., lower numbers). This holds within each treatment: homogenous ( $[A]$  to  $[A+]$ ), heterogenous ( $[B]$  to  $[B+]$ ) and replacement ( $[C]$  to  $[C+]$ ).

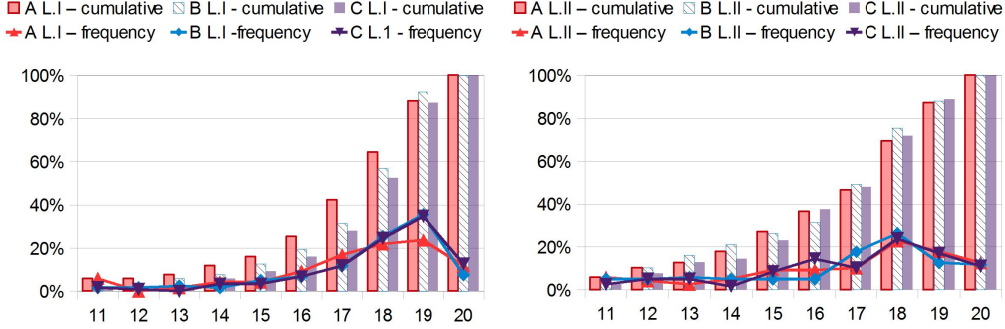


Figure 5: **Treatments [A], [B] and [C] for label  $I$  (left) and  $II$  (right):** Changing beliefs (comparison of treatments [A] and [B]) affects behavior in a way consistent with our model. Moreover, as predicted by our theory, higher-order beliefs effects (comparison of treatments [B] and [C]) are observed only for the more sophisticated subjects (label  $I$ ).

## 5 Five ‘Little Treasures’ of Game Theory

In this section we show that our model can be applied to make predictions across games, thereby shedding light on open empirical questions. In particular, we show that the predictions of our model are highly consistent with Goeree and Holt’s (2001, henceforth GH) well-known findings. In this influential paper, GH conduct a series of experiments on initial responses in different games. For each of these games, GH contrast individuals’ behavior in a baseline game, or ‘treasure’, with the behavior observed in a similar game, or ‘contradiction’, which differs only in the value of one parameter of the payoffs. GH show that classical equilibrium predictions often perform well in the treasure, but not in the contradiction. As GH report, existing models of strategic thinking are a useful first step in “organizing the qualitative patterns”, but they emphasize that “there are obvious discrepancies” (GH, p. 1418). As they, and others since, note, it is important to have a model that explains these intuitive patterns of behavior. But these results have been difficult to explain both qualitatively and quantitatively, particularly without making ad hoc assumptions for each game.

Our model provides a unified explanation for GH’s observed results. We argue that this explanation has qualitative appeal and show that it is highly predictive of GH’s data. In this analysis, we consider a version of the model with a single free parameter and calibrate that parameter using one of GH’s games. We then use this parameter, holding it fixed throughout, to predict behavior in GH’s other static games of complete information (the domain of our theory). We do not exploit any other degree of freedom, thereby further ensuring that our analysis does not make use of ad hoc assumptions. Comparing our predictions to the data reveals that our results are indeed strongly in line with GH’s findings.

Here we illustrate the logic behind the results, leaving the details of the quantitative analysis to Appendix C. We first review GH’s findings and briefly discuss why a classical level- $k$  approach does not suffice to explain them. We then present the results of our calibration.

## 5.1 Little Treasures: Review

**Matching Pennies.** Consider the following game, with payoffs parameterized by  $x > 40$ :

	L	R
T	$x, 40$	$40, 80$
B	$40, 80$	$80, 40$

With  $x = 80$ , this is a standard Matching Pennies game. Nash Equilibrium predicts that both the row and the column players mix uniformly over their two actions. Since  $x$  does not affect the payoffs of the column player, in any Nash Equilibrium the distribution over the row player's actions should be uniform independent of  $x$ . While the equilibrium prediction is in line with the data observed when  $x = 80$  (the 'treasure' treatment), when  $x = 320$  or  $x = 44$  (the 'contradiction' treatments), more than 95% of the row players choose the action with the relatively higher payoff:  $T$  when  $x = 320$  and  $B$  when  $x = 44$ . Moreover, this behavior seems to have been anticipated by some of the column players, with roughly 80% percent of subjects playing the best response to the action played by most of the row players, which is  $R$  when  $x = 320$  and  $L$  when  $x = 44$ .

**Coordination Game with a Secure Outside Option.** The following game, also parameterized by  $x$ , is a coordination game with one efficient and one inefficient equilibrium, which pay  $(180, 180)$  and  $(90, 90)$ , respectively. The column player also has a secure option  $S$  which pays 40 independent of the row player's choice.

	L	H	S
L	$90, 90$	$0, 0$	$x, 40$
H	$0, 0$	$180, 180$	$0, 40$

Notice that action  $S$  is dominated by a uniform distribution over  $L$  and  $H$ . Hence, changing  $x$  has no effect on the set of equilibria. However, GH's experimental data show that behavior is strongly affected by  $x$ . In the treasure treatment ( $x = 0$ ), a large majority of row and column players choose the efficient equilibrium action, and 80% of pairs coordinated on  $(H, H)$ . In the contradiction treatment ( $x = 400$ ), this percentage falls to 32%.

**Traveler's Dilemma.** In this version of Basu's (1994) well-known game, two players choose a number between 180 and 300 (inclusive). The reward they receive is equal to the lowest of their reports, but in addition the player who announces the higher number transfers a quantity  $x$  to the other player.<sup>23</sup> This game is dominance solvable for any  $x > 0$ , and 180 is the only equilibrium strategy. GH observe that, when  $x = 180$  (the 'treasure' treatment), roughly 80% choose numbers close to the Nash action, while when  $x = 5$  (the 'contradiction' treatment), roughly 80% of subjects choose numbers close to the highest claim.

<sup>23</sup>GH do not specify the rule in case of tie. We assume that there are no transfers in that case.

**Minimum-Effort Coordination Game.** Players in this game choose effort levels  $a_1$  and  $a_2$  which can be any integer between 110 and 170. Payoffs are such that  $u_i(a_1, a_2) = \min\{a_1, a_2\} - a_i \cdot x$ , where  $x$  is equal to 0.1 in one treatment and 0.9 in the other. Independent of  $x$ , any common effort level is a Nash equilibrium. The efficient equilibrium is the one with high effort. While the pure-strategy Nash equilibria are unaffected by this change in payoffs, GH’s experimental data show that agents exert lower effort when  $x$  is higher.

**Kreps Game.** The baseline and the modified games are described in the following table. The numbers in parenthesis represent the empirical distributions observed in the experiment:

Baseline:	<i>Left</i> (26)	<i>Middle</i> (8)	<i>Non Nash</i> (68)	<i>Right</i> (0)
<i>Top</i> (68)	200, 50	0, 45	10, 30	20, -250
<i>Bottom</i> (32)	0, -250	10, -100	30, 30	50, 40
Modified:	<i>Left</i> (24)	<i>Middle</i> (12)	<i>Non Nash</i> (64)	<i>Right</i> (0)
<i>Top</i> (84)	500, 350	300, 345	310, 330	320, 50
<i>Bottom</i> (16)	300, 50	310, 200	330, 330	350, 340

The modified game is obtained from the baseline simply by adding a constant of 300 to every payoff, which does not affect the equilibria. This game has two pure-strategy equilibria, (*Top, Left*) and (*Bottom, Right*), and one mixed-strategy equilibrium in which row randomizes between *Top* and *Bottom* and column randomizes between *Left* and *Middle*. Yet, a majority of column players choose the Non-Nash action. In this case as well, the change in payoffs has no effect on the column players, and only a small one on the row players.

The results of these experiments stand in sharp contrast with standard equilibrium concepts. Other heuristics, such as assuming that individuals play according to their ‘maxmin’ strategy, or based on risk or loss aversion, may explain the behavior observed in some games, but not in others. Concerning the classical level- $k$  approach, while it has convincingly demonstrated that individuals follow sequential reasoning processes, we have shown in the previous sections that it is important to endogenize the depth of reasoning when making predictions across games, particularly when incentives to reason vary, as in GH’s setting. The changes in incentives can impact individuals’ cognitive bounds and beliefs, and hence, the level according to which they play. This suggests that conducting a level- $k$  analysis without accounting for these factors in GH’s setting could be incomplete, which seems to be the case empirically. For instance, as GH point out, the results of the Traveler’s Dilemma would require an unusual distribution of levels (GH, p. 1417). Similarly, following the literature and assuming uniform anchors, to obtain findings roughly consistent with the Matching Pennies treasure and contradictions would require that 60 to 68 percent of subjects are level-2 and that there are essentially no level-3’s or higher, which is clearly at odds with classical findings.<sup>24</sup> In the case of the Coordination

<sup>24</sup>Alternatively, the 60 to 68 percent of subjects could consist of only even levels  $\geq 2$ , which is implausible. We maintain that anchors are uniform to avoid ad hoc assumptions.

Game with a Secure Option, fitting the data (even without imposing the same parameter as for Matching Pennies) would at best imply that the same percentage of coordination occurs, for the contradiction treatment, on  $(L, L)$  and  $(H, H)$ , which is not the case quantitatively and is contrary to the main qualitative insight for that game. Clearly, imposing the same parameters as for Matching Pennies worsens the fit.

The analysis that follows demonstrates the importance of accounting for variations in the depth of reasoning. The logic that drives the results is intuitive, further illustrating that this model is well-suited to explain GH’s findings.

## 5.2 Little Treasures: A Unified Explanation

A common feature of the GH’s treasures and contradictions is that the observed behavior appears intuitive and fundamentally linked to the nature of incentives. In all five games, the treasure and contradictions differ in a payoff parameter  $x$  which does not affect the (pure actions) best-reply functions. In the language of Section 2, this means that each treasure and its contradictions belong to the same cognitive equivalence class. Hence, we can use our model to understand the change of behavior by studying how varying the parameter  $x$  affects players’ incentives to reason, holding the costs of reasoning constant.<sup>25</sup>

To demonstrate that our analysis does not provide us with too much flexibility, we allow only one degree of freedom in the model. We then calibrate the single free parameter using one of GH’s games, and hold its value constant not only between a treasure and its contradictions, but throughout the games. Our predictions fit the empirical findings closely even with these stringent restrictions, thereby providing strong support for our theory.

We maintain the following assumptions: the anchors are uniformly distributed over  $A$ ; the cost functions are strictly increasing; and there are two types of players, one (strictly) more sophisticated than the other (in the sense of Def. 1), respectively denoted by ‘high’ and ‘low’. Let  $q_l$  denote the fraction of the low types. Fraction  $q_l$  is the parameter that we calibrate using one game and maintain as constant throughout all games and for both players. For identification purposes, we also assume that agents have correct beliefs over the distribution of types. Throughout this section we maintain that the value of reasoning takes the ‘maximum gain’ representation introduced in Example 2:

$$v_i(k) = \max_{a_j \in A_j} u_i(BR(a_j), a_j) - u_i(a_i^{k-1}, a_j). \tag{11}$$

We choose this functional form because it illustrates cleanly the logic of the model, and because it restricts the degrees of freedom available by completely fixing the value of reasoning. Other plausible representations, such as eq. (4) in Example 2, would allow for more degrees of freedom and would improve our estimates. Similarly, allowing for more types of sophistication

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<sup>25</sup>We do not make cognitive equivalence assumptions other than for each treasure and its respective contradictions. For instance, the costs of reasoning for Matching Pennies (and its contradictions) need not be the same as in the Traveler’s Dilemma (and its contradiction).

would also clearly improve the fit of our predictions to the data, without adding to the basic intuition.

**Qualitative Explanation of the Mechanism.** We use the Matching Pennies game described above to describe the way our model applies to GH’s games. Following the reasoning from Section 2.3.2, the low sophistication type plays according to his cognitive bound. We thus start by characterizing the cognitive bound of the low types, and hence their chosen action, before discussing the high types.

Consider first a low type of player 1, whose current action  $a_1^{k-1}$  in the path of reasoning is  $B$ . Using (11), his value of reasoning at step  $k$  is  $v_1(k) = x - 40$ . When instead  $a_1^{k-1}$  is  $T$ , his value of reasoning is  $v_1(k) = 40$ . As  $x$  increases, the value of reasoning increases if  $a_1^{k-1} = B$ , but *not* if  $a_1^{k-1} = T$ . In other words, there is an asymmetry in the incentives between having the action associated with  $k - 1$  being  $B$  or  $T$ . For a row player whose current action is  $B$ , sufficiently increasing  $x$  will lead him to perform one extra step of reasoning, and eventually stop at  $T$ . For a row player whose current level is  $T$ , the increase in  $x$  has no effect. Hence, for a sufficiently high increase in  $x$ , any low type player 1 will stop his reasoning at  $T$ . This does not depend on the anchor  $a^0$ . Hence, as  $x$  increases, either the low type player 1’s behavior stays the same, or (for sufficiently high  $x$ ) he plays  $T$ , independent of the anchor. Consider now the low type of player 2, who also plays according to his cognitive bound. Because  $x$  has no impact on his value of reasoning, his behavior does not change.<sup>26</sup>

Turning to the high types, their behavior depends not only on their cognitive bound, but also on their beliefs over the low types’ behavior. A high-type player 1 plays  $T$  if he believes a high fraction plays  $L$ , and a high-type player 2 chooses  $R$  if he believes a high enough fraction plays  $T$ . In essence, not only does the increase in  $x$  have an impact on the cognitive bound of player 1s, but it also has an effect on the high types’ beliefs over their opponents’ cognitive bound. This in turn affects their behavior in a predictable way: as with the low types, an increase in  $x$  either has no impact or (if large enough) it modifies their behavior in a sharply identifiable way.

To summarize, actions chosen for the low-type players depend only on their own cognitive bound, while those of the high-type players also depend on their beliefs over the low types’ play. Moreover, as payoffs are made asymmetric through the increase in  $x$ , incentives to reason are distorted. For high enough asymmetries, the anchor itself is no longer relevant: the behavior of the low types is driven by their incentives to reason, and becomes predictable. Depending on the parameter  $q_l$ , this in turn pins down the behavior of the high types.

Our calibration exercise is based on this logic, and uses the data from one game to identify  $q_l$ . We then use the calibrated parameter to predict the choices for the remaining games. We emphasize that the value of reasoning function is fully determined, and the only property of the cost functions used in this argument is that they are increasing. No further assumptions

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<sup>26</sup>This will be the case for both the Matching Pennies game and the Coordination Game with Secure Outside option. In the other games, the increase in  $x$  has an identical effect for player 1 and player 2.

<i>Data</i> (GH)			<i>Calibration</i> ( $q_t = 0.32$ )		
$x = 80$	$L(48)$	$R(52)$	$x = 80$	$L(50)$	$R(50)$
$T(48)$			$T(50)$		
$B(52)$			$B(50)$		
$x = 320$	$L(16)$	$R(84)$	$x = 320$	$L(16)$	$R(84)$
$T(96)$			$T(83)$		
$B(4)$			$B(17)$		
$x = 44$	$L(80)$	$R(20)$	$x = 44$	$L(84)$	$R(16)$
$T(2)$			$T(0)$		
$B(98)$			$B(100)$		

Table 3: Matching Pennies

<i>Data</i> (GH)				<i>Calibration</i> ( $q_t = 0.32$ )			
$x = 0$	$L(16)$	$H(84)$	$S$	$x = 0$	$L(17)$	$H(83)$	$S$
$L(4)$				$L(17)$			
$H(96)$				$H(83)$			
$x = 400$	$L$	$H$	$S$	$x = 400$	$L(33)$	$H(67)$	$S$
$L$	(16)			$L(49)$	(16)		
$H$		(32)		$H(51)$		(34)	

Table 4: Coordination Game with a Secure Outside Option

on the cost functions are needed.

**Results.** Tables 3 and 4 summarize the data from GH and the predictions of our model for Matching Pennies and the Coordination Game with a Secure Outside Option with parameter value  $q_t = 0.32$ . This parameter is calibrated on the Matching Pennies game with  $x = 320$ , and maintained throughout to make predictions on the remaining games.<sup>27</sup>

The data for the last three games (Kreps', Minimum-Effort Coordination and the Traveler's Dilemma) are fully consistent with the restrictions and calibrated parameter discussed above, but they do not require the full force of our assumptions.

For the Kreps game, for instance, the implications of our model are the simplest of all: since the modified game is identical to the baseline game plus an added constant of 300 to every payoff, Assumption 4 (Section 2) directly applies. Hence, the model predicts that whatever we observe in the baseline game should not change for the modified game. This prediction is close to the observed behavior, especially for the column players.<sup>28</sup>

<sup>27</sup>Besides the data summarized in the matrices, GH also report that 64 percent of row players and 76 of the column players play  $H$  in the contradiction treatment of the coordination game ( $x = 400$ ). These data however are inconsistent, probably due to a typographical error: if 76 percent of column play  $H$  and 32 percent of observations are  $(H, H)$ , then cell  $(L, H)$  must receive a weight of 44. Since  $(L, L)$  is observed 16 percent of the times, it follows that at least 60 percent of row played  $L$ , which is inconsistent with 64 percent of Row playing  $H$ .

<sup>28</sup>Note that this comparative statics holds under functional form (11), but also for the more general 'detail-free' model of Section 2.



In the case of the Minimum-Effort Coordination game, independent of the shape of the cost and benefit functions (provided  $c_i(1) < v_i(1)$ ), if the anchor is uniform, players of all types play 164 in the treasure and 116 in the contradiction. These results are close to the empirical findings, which are mainly concentrated near 170 and 110, respectively.

In the Traveler’s Dilemma, as  $x$  increases, Assumption 4 implies that the value of reasoning increases. Hence, by Proposition 2, individuals’ depth of reasoning would be higher in the high-reward treatment, and their chosen action would be lower. The observed change in behavior can therefore be explained by the stronger ‘incentives to reason’ that the game provides when  $x$  is increased from 5 to 180. The assumptions specified above, and the calibrated parameter for  $q_l$ , are entirely consistent with this result, but they are not necessary for this analysis. These parameters can serve, however, to enable a partial identification of the shape of the cost function. Identifying the cost of reasoning in different strategic settings is an important empirical question for future research.

## 6 Concluding Remarks

In this paper we have introduced a model of strategic thinking that endogenizes individuals’ cognitive bounds as the result of a cost-benefit analysis. Our theory distinguishes between players’ cognitive bounds and their beliefs about the opponent’s bound, and accounts for the interactions between depth of reasoning, incentives and higher-order beliefs. The tractability of the model has guided our experimental design to test these complex interactions.

From a theoretical viewpoint, we extend the general level- $k$  approach of taking reasoning in games to be procedural and possibly constrained. By making explicit these appealing features of level- $k$  models, our framework serves to attain a deeper understanding of the underlying mechanisms of that approach. Our framework also solves apparent conceptual difficulties of the level- $k$  approach, such as the possibility that individuals reason about opponents they regard as more sophisticated. In addition to testing the model, our experiment plays a broader role. It reveals that individuals change their behavior in a systematic way as their incentives and beliefs are varied. Thus, caution should be exercised in interpreting level of play as purely revealing of cognitive ability, as an endogeneity problem is present.<sup>29</sup> Our model serves as a natural and tractable candidate to address this endogeneity problem and provides a unified theory of procedural rationality in strategic settings.

Using a calibration exercise, we have shown that the predictions of our model are highly consistent with the empirical findings of Goeree and Holt’s (2001) influential ‘little treasures’ experiments. This provides further support for our theory and an external validation of the approach. Since Goeree and Holt’s games have a very different structure from those of our experiment, this exercise also shows that our theory is applicable to a wide range of games.

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<sup>29</sup>In a different setting, it is a well-known theme in the Economics of Education literature that incentives may affect standard measures of cognitive abilities. For a recent survey of the vast literature that combines classical economic notions with measurement of cognitive abilities and psychological traits to address the endogeneity problems stemming from the role of incentives, see Almlund, Duckworth, Heckman and Kautz (2011).

In closing, we note that our theory establishes a link between level- $k$  reasoning and the conventional domain of economics, centered around tradeoffs and incentives. From a methodological viewpoint, this can further favor the integration of theories of initial responses within the core of economics. Conversely, the application of classical economic concepts to a model of reasoning opens new directions of research both theoretically and empirically. For instance, future research could include a rigorous identification of the properties of cost functions in different games and testing predictions of changes in behavior across other strategic settings.

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# Appendix

## A Logistics of the Experiment

The experiment was conducted at the Laboratori d'Economia Experimental (LEEX) at Universitat Pompeu Fabra (UPF), Barcelona. Subjects were students of UPF, recruited using the LEEX system. No subject took part in more than one session. Subjects were paid 3 euros for showing up (students coming from a campus that was farther away received 4 euros instead). Subjects' earnings ranged from 10 to 40 euros, with an average of 15.8.

Each subject went through a sequence of 18 games. Payoffs are expressed in 'tokens', each worth 5 cents. Subjects were paid randomly, once every six iterations. The order of treatments is randomized (see below). Finally, subjects only observed their own overall earnings at the end, and received no information concerning their opponents' results.

Our subjects were divided in 6 sessions of 20 subjects, for a total of 120 subjects. Three sessions were based on the exogenous classification, and each contained 10 students from the field of humanities (humanities, human resources, and translation), and 10 from math and sciences (math, computer science, electrical engineering, biology and economics). Three sessions were based on the endogenous classification, and students were labeled based on their performance on a test of our design. (See Appendix D). In these sessions, half of the students were labeled as 'high' and half as 'low'.

### A.1 Instructions of the Experiment

We describe next the instructions as worded for a student from math and sciences. The instructions for students from humanities would be obtained replacing these labels everywhere. Similarly, labels high and low would be used for the endogenous classification.

#### A.1.1 Baseline Game and Treatments [A], [B] and [C]

Pick a number between 11 and 20. You will always receive the amount that you announce, in tokens.

In addition:

- if you give the same number as your opponent, you receive an extra 10 tokens.
- if you give a number that's exactly one less than your opponent, you receive an extra 20 tokens.

*Example:*

- If you say 17 and your opponent says 19, then you receive 17 and he receives 19.
- If you say 12 and your opponent says 13, then you receive 32 and he receives 13.
- If you say 16 and your opponent says 16, then you receive 26 and he receives 26.

**Treatments [A] and [B]:**

Your opponent is:

- a student from maths and sciences (treatment [A]) / humanities (treatment [B])
- he is given the same rules as you.

### **Treatment [C]:**

In this case, the number you play against is chosen by:

- a student from humanities facing another student from humanities. In other words, two students from humanities play against each other. You play against the number that one of them has picked.

### **A.1.2 Changing Payoffs: Treatments [A+], [B+] and [C+]**

You are now playing a high-payoff game. Pick a number between 11 and 20. You will always receive the amount that you announce, in tokens.

In addition:

- if you give the same number as your opponent, you receive an extra 10 tokens.
- if you give a number that's exactly one less than your opponent, you receive an extra 80 tokens.

*Example:*

- If you say 17 and your opponent says 19, then you receive 17 and he receives 19.
- If you say 12 and your opponent says 13, then you receive 92 and he receives 13.
- If you say 16 and you opponent says 16, then you receive 26 and he receives 26.

### **Treatments [A+] and [B+]**

Your opponent is:

- a student from maths and sciences playing the high-payoff game (treatment [A+]) / humanities (treatment [B+])
- he is given the same rules as you.

### **Treatment [C+]**

In this case, the number you play against is chosen by:

- a student from humanities playing the high payoff game with another student from humanities. In other words, two students from humanities play the high payoff game with each other (extra 10 if they tie, 80 if exactly one less than opponent). You play against the number that one of them has picked.

## **A.2 Sequences**

Our 6 groups (3 for the endogenous and 3 for the exogenous classification) went through four different sequences of treatments. Two of the groups in the exogenous treatment followed Sequence 1, and one followed Sequence 2. The three groups of the endogenous classification each took a different sequence: respectively sequence 1, 3 and 4. All the sequences contain our main treatments, [A], [B], [C], [A+], [B+], [C+]. The order of the main treatments is different in each sequence, both in terms of changing the beliefs and the payoffs. (These sequences include additional treatments [K], [L], [D], [E] and [F] discussed in the working paper.)

- **Sequence 1:**  $A, B, C, B, A, C, A^+, B^+, C^+, B^+, A^+, C^+, D, E, F, D, E, F$
- **Sequence 2:**  $A, B, B, A, C, C, K, L, K, L, K, L, A^+, B^+, B^+, A^+, C^+, C^+$
- **Sequence 3:**  $A^+, B^+, C^+, B^+, A^+, C^+, A, B, C, B, A, C, D, E, F, D, E, F$
- **Sequence 4:**  $B, A, C, A, B, C, K, L, K, L, K, L, B^+, A^+, C^+, A^+, B^+, C^+$

## B Statistical Tests and Regressions

	Relevant dummy	Classification dummy	Constant	Number of obs.
From A to A+ Label <i>I</i>	-0.50*** (0.17)	0.22 (0.52)	17.21	235
From B to B+ Label <i>I</i>	-0.62*** (0.19)	0.36 (0.38)	17.50	233
From C to C+ Label <i>I</i>	-1.15*** (0.18)	0.34 (0.38)	17.76	236
From A to A+ Label <i>II</i>	-0.64** (0.27)	-0.10 (0.45)	16.91	236
From B to B+ Label <i>II</i>	-0.74*** (0.25)	0.38 (0.47)	16.57	236
From C to C+ Label <i>II</i>	-0.97*** (0.25)	-0.07 (0.45)	16.97	234
From B to A Label <i>I</i>	-0.36* (0.20)	0.34 (0.43)	17.50	236
From B to C Label <i>I</i>	0.25 (0.18)	0.44 (0.37)	17.46	236
From A to C Label <i>I</i>	0.62*** (0.18)	0.36 (0.42)	17.13	236
From A to B Label <i>II</i>	-0.09 (0.26)	0.50 (0.48)	16.6	236
From B to C Label <i>II</i>	0.16 (0.25)	0.36 (0.46)	16.59	235
From A to C Label <i>II</i>	0.07 (0.27)	0.30 (0.44)	16.71	235
From B+ to A+ Label <i>I</i>	-0.26 (0.17)	0.25 (0.46)	16.96	232
From B+ to C+ Label <i>I</i>	-0.29** (0.13)	0.27 (0.45)	16.93	233
From A+ to C+ Label <i>I</i>	0.03 (0.16)	0.21 (0.48)	16.71	236
From A+ to B+ Label <i>II</i>	-0.18 (0.21)	-0.22 (0.52)	16.33	236
From B+ to C+ Label <i>II</i>	-0.07 (0.21)	-0.04 (0.50)	16.05	235
From A+ to C+ Label <i>II</i>	-0.25 (0.22)	-0.47 (0.51)	16.46	235

Table 5: Regressions for Labels *I* and *II*. \* indicates < 10% significance, \*\* indicates < 5% significance and \*\*\* indicates < 1% significance. Standard errors in parenthesis. The ‘Classification Dummy’ refers to the exogenous vs endogenous criterion.



	Two-Sample Kolmogorov-Smirnov D-stat (exact p-value)	Wilcoxon-signed-rank p-values
A vs A+ Label <i>I</i>	0.18 (0.040) **	0.0002 ***
B vs B+ Label <i>I</i>	0.24 (0.002) ***	0.0001 ***
C vs C+ Label <i>I</i>	0.37 (0.000) ***	0.0000 ***
A vs A+ Label <i>II</i>	0.14 (0.128)	0.0020 ***
B vs B+ Label <i>II</i>	0.17 (0.048) **	0.0013 ***
C vs C+ Label <i>II</i>	0.25 (0.001) ***	0.0001 ***

Table 6: Equality of Distributions Tests: Changing Payoffs. \* indicates < 10% significance, \*\* indicates < 5% significance and \*\*\* indicates < 1% significance.

	Two-Sample Kolmogorov-Smirnov D-stat (exact p-value)	Wilcoxon-signed-rank p-values
A vs B Label <i>I</i>	0.11 (0.463)	0.37
B vs C Label <i>I</i>	0.05 (0.998)	0.37
A vs C Label <i>I</i>	0.14 (0.163)	0.02 **
A+ vs B+ Label <i>I</i>	0.06 (0.952)	0.08 *
B+ vs C+ Label <i>I</i>	0.08 (0.691)	0.03 **
A+ vs C+ Label <i>I</i>	0.07 (0.833)	0.82
A vs B Label <i>II</i>	0.05 (0.986)	0.88
B vs C Label <i>II</i>	0.06 (0.930)	0.73
A vs C Label <i>II</i>	0.04 (1.000)	0.79
A+ vs B+ Label <i>II</i>	0.07 (0.793)	0.17
B+ vs C+ Label <i>II</i>	0.06 (0.938)	0.23
A+ vs C+ Label <i>II</i>	0.12 (0.277)	0.08 *

Table 7: Equality of Distributions Tests: Changing Opponents. \* indicates < 10% significance, \*\* indicates < 5% significance and \*\*\* indicates < 1% significance.

## C Calibration

### C.1 Matching Pennies

Given the functional form in (11), the value of reasoning is:

$$v_1(k) = \begin{cases} 40 & \text{if } a_1^{k-1} = T \\ x - 40 & \text{if } a_1^{k-1} = B \end{cases} \quad (12)$$

$$v_2(k) = 40.$$

The paths of reasoning have a periodicity of 4. For instance, for  $a^0 = (B, L)$ , the path is  $(B, L), (T, L), (T, R), (B, R), (B, L), \dots$ . The cases for the other three possible anchors are obtained similarly.

Fix cost functions  $(c^l, c^h)$ , and let  $x = 80$ . Then,  $v_i(k) = 40$  for all  $i$  and  $k$ . Let  $v^{80}$  denote such a function. Define  $k^h = K(c^h, v^{80})$ ,  $k^l = K(c^l, v^{80})$  and  $\Delta k := k^h - k^l$ . Under the maintained assumptions,  $\Delta k \geq 1$ . Given the symmetry of the incentives and the uniform distribution of the anchors, with  $x = 80$  we obtain a uniform distribution over actions in both populations, independent of the value of  $q_l$ . Hence, when  $x = 80$ , actions are uniformly distributed in both populations, as predicted by the unique mixed-strategy equilibrium. It is clear, however, that “equilibrium play in this case is attained only by coincidence” (cf. GH, p. 1407): with symmetric incentives (across players and across actions), behavior is completely driven by the anchors.

Now, suppose that  $x$  is increased above 80, and consider the low types of population 1. For these players, the value of reasoning is no longer constant:  $v_1(k+1)$  now alternates between 40 and  $x - 40 > 40$ , depending on whether the current action is  $a_1^k = B$  or  $a_1^k = T$ . Players whose cognitive bound with  $x = 80$  was such that  $a_1^{\hat{k}_i^{80}} = T$  see no change in the value of the next step of reasoning. Their depth of reasoning therefore does not change either. For players who had stopped at  $a_i^{\hat{k}_i^{80}} = B$  instead, the value of the next step is now higher. Hence, they would perform an extra step if and only if  $x - 40 > c_l(\hat{k}_i^{80} + 1)$ .<sup>30</sup> Summarizing, for any pair of increasing functions  $(c^l, c^h)$ , if  $x$  is increased but below a certain threshold, then behavior does not change. Above that threshold, all the low types of population 1 play  $T$ , the behavior of low types in population 2 remains the same, and behavior of the high types depends on  $q_l$ .

We now consider the case  $x = 320$ . Since behavior is different from the case in which  $x = 80$ , by the logic above we assume that  $x$  is sufficiently high that all agents in population 1 stop their reasoning at a point in which they consider action  $T$ . With these payoffs, action  $T$  is a best response to any beliefs that attach probability 1/8 or higher to the opponents playing  $L$ . The cut-off probability for population 2 instead remains at 1/2.

1. Suppose that  $q_l < 1/8$ . When  $q_l < 1/8$ , the behavior of the low types is not enough to pin down the behavior of the high types in either population. It follows that all types will play at

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<sup>30</sup>These players perform one or two extra steps, and no more, because  $v(\hat{k}_i^{80} + 3) = 40 < c_l(\hat{k}_i^{80} + 1) < c_l(\hat{k}_i^{80} + 2) < c_l(\hat{k}_i^{80} + 3)$ . The first inequality follows from the definition of  $\hat{k}_i^{80}$ , and the others from  $c_l$  being increasing.

their cognitive bound. In particular, this implies that the actions of both types in population 2 are uniformly distributed. The prediction that population 2 plays uniformly with  $x = 320$  is inconsistent with the data. We thus rule out the case  $q_l < 1/8$ .

2. Suppose that  $q_l > 1/2$ . Since, independent of the anchor, all low types of population 1 switch to  $T$ , then all high types of population 2 play  $R$  if  $q_l > 1/2$ . The low types of population 2 instead are uniformly distributed, following their own cognitive bound. It follows that a fraction  $\frac{q_l}{2} + (1 - q_l)$  of population 2 plays  $R$ . Given the uniformity assumption on the anchors, half of the high types in population 1 believe that the low types of population 2 play  $L$ , and since  $q_l > 1/8$ , they play  $T$ . The remaining half of high types of population 1 believe the low types of population 2 play  $R$ , hence they play  $B$ . The resulting distribution for  $q_l > 1/2$  therefore is such that a fraction  $(\frac{1}{2} + \frac{q_l}{2})$  of player 1s play  $T$ , and a fraction  $(\frac{q_l}{2})$  of player 2s play  $L$ .
3. Suppose that  $q_l \in (1/8, 1/2)$ . Given  $c_l$ , depending on what the anchor is, we may have the following cases:

- (a)  $a^{k^l} = (B, L)$  or  $a^{k^l} = (T, L)$ . In this case, which applies to half of the population, the low types of population 2 play  $L$ . Since  $q_l > 1/8$ , this is enough to convince the high types of population 1 to play  $T$ . The low types of population 1 play  $T$ , because their increased incentives moved their cognitive bound to  $T$ .
  - i. If  $\Delta k \geq 2$ , then the high types of population 2 understand everything thus far, hence play  $R$ .
  - ii. If  $\Delta k = 1$ , then the high types of population 2 are not sufficiently ‘deep’ to understand the choice of the high types of population 1 (which are best responding to the low types of population 2). They thus play at their bound. Whether this is  $L$  or  $R$  depends on the anchor being  $(B, L)$  or  $(B, R)$ , which is uniformly distributed.
- (b)  $a^{k^l} = (B, R)$ . In this case, which applies to a quarter of the population, the low types of population 2 play  $R$  and the low types of population 1 play  $T$  (because of the increased incentives, they stop at step  $k^l + 1$ ). Since  $q_l \in (1/8, 1/2)$ , these are not enough to pin down the behavior of the high types in either population. The high types of both populations therefore play at their bound, which is  $(B, L)$  if  $\Delta k = 1 \pmod{4}$ ,  $(T, L)$  if  $\Delta k = 2 \pmod{4}$ ,  $(T, R)$  if  $\Delta k = 3 \pmod{4}$  and  $(B, R)$  if  $\Delta k = 4 \pmod{4}$ .
- (c)  $a^{k^l} = (T, R)$ . In this case, which applies to a quarter of the population, the low types of population 2 play  $R$  and the low types of population 1 play  $T$ . Since  $q_l \in (1/8, 1/2)$ , these are not enough to pin down the behavior of the high types in either population. The high types of both populations therefore play at their bound, which is  $(B, R)$  if  $\Delta k = 1 \pmod{4}$ ,  $(B, L)$  if  $\Delta k = 2 \pmod{4}$ ,  $(T, L)$  if  $\Delta k = 3 \pmod{4}$  and  $(T, R)$  if  $\Delta k = 4 \pmod{4}$ .

Aggregating cases (a), (b) and (c), we have the following possibilities for  $q_l \in (1/8, 1/2)$ : if  $\Delta k = 1$ , then  $T$  is played by fraction  $(\frac{1}{2} + \frac{q_l}{2})$  of player 1s and  $L$  by  $1/2$  of player 2s. If instead  $\Delta k \geq 2$ , then we have the following cases: (i)  $\Delta k = 1 \pmod{4}$ , in which  $T$  is played by fraction  $(\frac{1}{2} + \frac{q_l}{2})$

of players 1 and  $L$  by a fraction  $(\frac{1}{4} + \frac{q_l}{4})$  of player 2s; (ii)  $\Delta k = 2 \pmod{4}$ , in which  $T$  is played by fraction  $(\frac{3}{4} + \frac{q_l}{4})$  of player 1s and  $L$  by a  $1/2$  of player 2s; (iii)  $\Delta k = 3 \pmod{4}$ , in which  $T$  is played by all player 1s and  $L$  by a fraction  $(\frac{1}{4} + \frac{q_l}{4})$  of player 2; (iv)  $\Delta k = 4 \pmod{4}$ , in which  $T$  is played by fraction  $(\frac{3}{4} + \frac{q_l}{4})$  of players 1 and  $L$  by a fraction  $(\frac{q_l}{2})$  of player 2s.

As above, we can discard cases  $\Delta k = 1$  and  $\Delta k = 2 \pmod{4}$  based on the observation that the distribution of actions in population 2 is not uniform. With  $q_l \geq 1/8$ , cases  $\Delta k = 1 \pmod{4}$ ,  $3 \pmod{4}$  entail at least 37.5% of population 2 playing  $L$ . The data only show a 16%, hence we discard this possibility as well. The only case that is left therefore is  $\Delta k = 4 \pmod{4}$ , which yields the same distribution of actions for population 2 as case 2 above (that is,  $q_l/2$ ).

Overall, we are left with two possibilities, both entailing that a fraction  $q_l/2$  of population 2 plays  $L$ . Choosing  $q_l$  to match the empirical distributions, we obtain  $q_l = 0.32$ , which falls precisely in the interval  $(1/8, 1/2)$ . Therefore the only explanation that appears consistent with the empirical distribution of population 2 is the following, which corresponds to the case  $\Delta k = 4 \pmod{4}$ :

Calibration:  $q_l = 0.32$  [Data in Brackets]

$x = 320$	$L$ (16)*	$R$ (84)*
$T$ (83) [96]		
$B$ (17) [4]		

We next consider the case with  $x = 44$ , maintaining that  $q_l \in (1/8, 1/2)$  from the previous exercise. Notice that for this game, player 1 plays  $B$  as soon as he attaches probability at least  $1/11$  on  $R$  being played. First, it is easy to show that for any increasing cost functions  $c_l, c_h$ , there exists  $x > 40$  sufficiently low that the both types of population 1 would choose  $B$  at their cognitive bound. Assuming that  $x = 44$  is ‘sufficiently low’, a reasoning similar to the above delivers the following results: all low types of population 1 play  $B$ , while the low types of population 2 are uniformly split; if  $q_l \in (1/8, 1/2)$ , the 50% of high types of population 1 that believe that the low types of population 2 play  $R$  will play  $B$  (because  $q_l > 1/8 > 1/11$ ), and the 50% of high types in population 2 that anticipate this will play  $L$ . The remaining 50% of high types in population 1 play according to their own cognitive bound, that is  $B$ . Since the high types in population 2 have the same cost function, but higher incentives, they would be able to anticipate this, and respond playing  $L$ .

Summarizing, for  $q_l \in (1/8, 1/2)$  our findings for the three games are:

$x = 80$	$L(\frac{1}{2})$	$R(\frac{1}{2})$	$x = 320$	$L$ ( $\frac{q_l}{2}$ )	$R$ ( $1 - \frac{q_l}{2}$ )	$x = 44$	$L(1 - \frac{q_l}{2})$	$R(\frac{q_l}{2})$
$T$ ( $\frac{1}{2}$ )			$T$ ( $\frac{3}{4} + \frac{q_l}{4}$ )			$T$ (0)		
$B$ ( $\frac{1}{2}$ )			$B$ ( $\frac{1}{4} - \frac{q_l}{4}$ )			$B$ (1)		

## C.2 Coordination Game with a Secure Option

This game has two pure-strategy Nash equilibria,  $(L, L)$  and  $(H, H)$ , which are not affected by the value of  $x$ . Hence, anchors equal to  $(L, L)$  or  $(H, H)$  would generate a path of reasoning in which respectively  $L$  or  $H$  is repeated. Anchors  $(L, H)$  or  $(H, L)$  determine a cycle alternating between

$H$  and  $L$ , which is also independent on the value of  $x$ . The paths generated by anchors that involve  $S$ , instead, vary with the value of  $x$ , but since action  $S$  is dominated, it is never part of any path of reasoning for any  $k > 0$ . Nonetheless, it shapes player 1's incentives to reason, as an increase in  $x$  changes the value of doing a step of reasoning when player 1 is in a state in which action  $H$  is regarded as the most sophisticated. Applying equation (11) to this game, with payoffs parameterized by  $x$ , we obtain the following value of reasoning functions:

$$v_1(k) = \begin{cases} 180 & \text{if } a_1^{k-1} = L \\ \max\{90, x\} & \text{if } a_1^{k-1} = H \end{cases} \quad (13)$$

$$v_2(k) = \begin{cases} 90 & \text{if } a_2^{k-1} = H \\ 180 & \text{if } a_2^{k-1} = L \\ 140 & \text{if } a_2^{k-1} = S \end{cases} \quad (14)$$

Similar to the asymmetric matching pennies games discussed above, any path in which agents cycle between action  $L$  and action  $H$  induces a  $v_1$  function that alternates between 90 and 180. Whether the spikes are associated to odd or even  $k$ 's depends on the anchor. When  $x = 400$ , the incentives to reason do not change for player 2, but  $v_1$  changes, alternating between 180 and 400: the 'spikes' at 400 replace what would be 'troughs' at 90 with  $x = 0$ .

The experimental results show that 96% of player 1s and 84% of player 2s played  $H$  when  $x = 0$ .<sup>31</sup> Under the assumption that anchors are uniformly distributed, the only way that such a strong coordination on  $H$  can be explained is by assuming that the 'spikes' and 'troughs' determined alternating between 180 and 90 are already sufficiently pronounced that the types involved in a reasoning process that determines a cycle stop their reasoning at  $H$ . Hence, with  $x = 0$ , agents that approach the game with anchors  $a^0 = (L, L)$  play  $L$ , all others play  $H$  (because they either settle on a constant  $H$ , as in  $a^0 = (H, S), (H, H)$ , or they determine a cycle, as in  $a^0 = (H, L), (L, H), (L, S)$ ). The predictions of the model therefore are the following:

$x = 0$	L (1/6)	H (5/6)	S (0)
L (1/6)			
H (5/6)			

We next consider the case  $x = 400$ , maintaining the assumption that the anchors are uniformly distributed. For the same reasons discussed above, for any pair of (increasing) cost functions  $c_l, c_h$ , there exists an  $x$  sufficiently high that all low types of population 1 with a reasoning process that involves a cycle stop at  $L$ . If  $q_l < 2/3$ , however, this is not enough to induce the high types of population 2 to play  $L$  as well. Hence, with  $q_l = 0.32$  as calibrated above, both the low and the high types in population 2 play according to their own cognitive bound. Since the incentives to reason were not affected by the

<sup>31</sup>One possible explanation is that in the baseline coordination game the efficient equilibrium is sufficiently focal that most individuals approach the game with  $a^0 = (H, H)$  as an anchor. While we think this is a plausible explanation, we explore here to what extent the mere change in incentives may explain the observed variation in behavior, independent of the possible change in the anchors. We note that assuming that the anchor is the uniform distribution delivers very similar quantitative results.

change in  $x$  for these individuals, the assumptions above imply that they play  $H$ . Hence, in population 2, all individuals with anchors  $a^0 \neq (L, L), (L, S)$  play  $H$ , and the others play  $L$ . It remains to consider the high types of population 1. Since with  $x = 400$  these types have stronger incentives to reason than the high types of population 2, any of these types involved in a cycle anticipates that both types of population 2 would play  $H$ , hence they respond with  $H$ . Thus, in population 1, only the individuals whose anchor is  $a^0 = (H, H)$  and the high types with anchors  $a^0 \neq (L, L), (L, S)$  play  $H$ , that is a total of  $1/6 + \frac{(1-q_l)}{2}$ , or  $2/3 - q_l/2$ . The others play  $L$ . To determine the percentages of coordination in  $(L, L)$  and  $(H, H)$ , we assume independence in the distributions of play between the row and the column players.

Summarizing:

$x = 400$	L (1/3)	H (2/3)	S (0)
L ( $1/3 + q_l/2$ )			
H ( $2/3 - q_l/2$ )			

with  $q_l = .32$  calibrated from the matching pennies game:

$x = 400$	L (33)	H (67)	S (0)
L (49)	16		
H (51)		34	

# Online Appendix

## D The Test for the Endogenous Classification

The cognitive test takes roughly thirty minutes to complete, and consists of three questions. In the first, subjects are asked to play a variation of the board game Mastermind. In the second question, the subjects are given a typical centipede game of seven rounds, and are asked what an infinitely sophisticated and rational agent would do. In the third game, the subjects are given a lesser known ‘pirates game’, which is a four player game that can be solved by backward induction. Subjects are asked what the outcome of this game would be, if players were ‘infinitely sophisticated and rational’. Each question was given a score, and then a weighted average was taken. Subjects whose score was higher (lower) than the median score were labeled as ‘high’ (‘low’). We report next the instructions of the test, as administered to the students (see the online appendix for the original version in Spanish).

**Instructions of the Test.** This test consists of three questions. You must answer all three within the time limit stated.

### Question 1:

In this question, you have to guess four numbers in the correct order. Each number is between 1 and 7. No two numbers are the same. You have nine attempts to guess the four numbers. After each attempt, you will be told the number of correct answers in the correct place, and the number of correct numbers in the wrong place.

*Example:* Suppose that the correct number is: 1 4 6 2.

If you guess : 3 5 4 6, then you will be told that you have 0 correct answers in the correct place and 2 in the wrong place.

If you guess : 3 5 6 4, then you will be told that you have 1 correct answer in the correct place and 1 in the wrong place.

If you guess : 3 4 7 2, then you will be told that you have 2 correct answers in the correct place and 0 in the wrong place.

If you guess : 1 4 6 2, then you will be told that you have 4 correct answers, and you have reached the objective.

Notice that the correct number could not be (for instance) 1 4 4 2, as 4 is repeated twice. You are, however, allowed to guess 1 4 4 2 , in any round.

You have a total of 90 second per round: 30 seconds to introduce the numbers and 60 seconds to view the results.

### Question 2:

Consider the following game. Two people, Antonio and Beatriz, are moving sequentially. The game starts with 1 euro on the table. There at most 6 rounds in this game:

*Round 1)* Antonio is given the choice whether to take this 1 euro, or pass, in which case the game has another round. If he takes the euro, the game ends. He gets 1 euro, Beatriz gets 0 euros. If Antonio passes, they move to round 2.

*Round 2)* 1 more euro is put on the table. Beatriz now decides whether to take 2 euros, or pass. If she takes the 2 euros, the game ends. She receives 2 euros, and Antonio receives 0 euros. If Beatriz passes, they move to round 3.

*Round 3)* 1 more euro is put on the table. Antonio is asked again: he can either take 3 euros and leave 0 to Beatriz, or pass. If Antonio passes, they move to round 4.

*Round 4)* 1 more euro is put on the table. Beatriz can either take 3 euros and leave 1 euro to Antonio, or pass. If Beatriz passes, they move to round 5.

*Round 5)* 1 more euro is put on the table. Antonio can either take 3 euros and leave 2 to Beatriz, or pass. If Antonio passes, they move to round 6.

*Round 6)* Beatriz can either take 4 euros and leaves 2 to Antonio, or she passes, and they both get 3.

Assume Antonio and Beatriz are infinitely sophisticated and rational and they each want to get as much money as possible. What will be the outcome of the game?

- a) Game stops at Round 1, with payoffs: (Antonio: 1 euro      Beatriz: 0 euros)
- b) Game stops at Round 2, with payoffs: (Antonio: 0 euro      Beatriz: 2 euros)
- c) Game stops at Round 3, with payoffs: (Antonio: 2 euros      Beatriz: 1 euro)
- d) Game stops at Round 4, with payoffs: (Antonio: 1 euro      Beatriz: 3 euros)
- e) Game stops at Round 5, with payoffs: (Antonio: 3 euros      Beatriz: 2 euros)
- f) Game stops at Round 6, with payoffs: (Antonio: 2 euros      Beatriz: 4 euros)
- g) Game stops at Round 6, with payoffs: (Antonio: 3 euros      Beatriz: 3 euros)

You have 8 minutes in total for this question.

**Question 3:**

Four pirates (Antonio, Beatriz, Carla and David) have obtained 10 gold doblónes and have to divide up the loot. Antonio proposes a distribution of the loot. All pirates vote on the proposal. If half the crew or more agree, the loot is divided as proposed by Antonio.

If Antonio fails to obtain support of at least half his crew (including himself), then he will be killed. The pirates start over again with Beatriz as the proposer. If she gets half the crew (including herself) to agree, then the loot is divided as proposed. If not, then she is killed, and Carla then makes the proposal. Finally, if her proposal is not agreed on by half the people left, including herself, then she is killed, and David takes everything.

In other words:

Antonio needs 2 people (including himself) to agree on his proposal, and if not he is killed.

If Antonio is killed, Beatriz needs 2 people (including herself) to agree on her proposal, if not she is killed.

If Beatriz is killed, Carla needs 1 person to agree (including herself) to agree on her proposal, and if not she is killed.

If Carla is killed, David takes everything.

The pirates are infinitely sophisticated and rational, and they each want to get as much money as possible. What is the maximum number of coins Antonio can keep without being killed?



Notice that \*the proposer\* can also vote, and that exactly half the votes is enough for the proposal to pass.

You have 8 minutes in total for this question.

**Scoring.** In the *mastermind* question, subjects were given 100 points if correct, otherwise they received 15 points for each correct answer in the correct place and 5 for each correct answer in the wrong place in their last answer. In the *centipede* game, subjects were given 100 points if they answered that the game would end at round 1, otherwise points were equal to  $\min\{0, (6 - \text{round}) \cdot 15\}$ . In the *pirates* game, subjects obtain 100 if they answer 100, 60 if they answer 10, and  $\min\{0, (80 - x) \cdot 10\}$  otherwise. The overall score was given by the average of the three.

## E Additional Observations

A vs A+	11	12	13	14	15	16	17	18	19	20
11	8				1	1		2	2	
12	2							2	1	
13		2				2			1	
14		1		2	1	3	1	1	2	
15		1		6	4		3	2		
16	1	2		3	2	7	3	3	1	
17	2			3	1	7	9	7	3	
18	2	2	2	1	2	7	16	13	5	2
19	3				1	4	11	22	6	2
20		1			1	1	7	4	5	10

changes in the direction consistent with the theory: 78%

B vs B+	11	12	13	14	15	16	17	18	19	20
11	2	2	1	1	1			1		
12	1		2					1	2	2
13	2			1	2	2	1	1	1	
14	1	1		2	1		1	2		
15			1	1	4	3	2	1		
16		1	1		1	4	5	1	1	
17	3	1	2	2	3	4	8	8	3	1
18	1	1	3	1	6	6	20	18	2	
19	2		1		1		14	20	16	3
20	1					5	6	8	2	1

changes in the direction consistent with the theory: 76%

C vs C+	11	12	13	14	15	16	17	18	19	20
11	2					1	1		1	
12		1		1	1		1	3		
13	1				1	1	2	1		
14		1		3	1			1		
15	2		1		5	2	3	1		
16	1	1	2	2	4	10	4	1		
17	2	3		2	3	7	9			
18	1	1	1	4	3	5	13	22	4	3
19	4	1	1	1	1	8	14	24	4	3
20	2				1	3	5	8	5	4

changes in the direction consistent with the theory: 84%

Figure 6: Transition Matrices

payoff functions							
	A.L.I	B.L.I	C.L.I	A+ L.I	B+ L.I	C+ L.I	
11	<b>11.59</b>	<b>12.53</b>	<b>12.44</b>	13.74	<b>15.83</b>	15.92	
12	12.34	13.69	12.93	<b>12.94</b>	16.58	<b>13.19</b>	
13	14.02	14.61	14.27	15.14	18.25	21.22	
14	15.27	15.53	16.37	18.36	22.05	19.76	
15	17.29	16.53	17.80	26.45	24.75	26.44	
16	20.32	20.07	18.97	37.88	35.41	30.92	
17	23.10	<b>24.03</b>	22.59	<b>39.39</b>	<b>38.95</b>	<b>37.00</b>	
18	<b>24.95</b>	23.17	<b>23.85</b>	27.32	26.56	31.14	
19	23.75	22.64	23.32	27.38	20.44	22.39	
20	21.19	21.19	21.27	20.94	20.08	20.25	

	A.L.II	B.L.II	C.L.II	A+ L.II	B+ L.II	C+ L.II	
11	<b>12.44</b>	<b>11.51</b>	<b>11.59</b>	15.92	<b>11.35</b>	13.74	
12	12.93	12.68	12.34	<b>13.19</b>	15.48	<b>12.94</b>	
13	14.27	13.59	14.02	21.22	14.13	15.14	
14	16.37	15.19	15.27	19.76	19.65	18.36	
15	17.80	16.86	17.29	26.44	23.35	26.45	
16	18.97	19.05	20.32	30.92	39.22	37.88	
17	22.59	23.27	23.10	<b>37.00</b>	<b>42.04</b>	<b>39.39</b>	
18	<b>23.85</b>	<b>27.66</b>	<b>24.95</b>	31.14	33.30	27.32	
19	23.32	24.08	23.75	22.39	23.35	27.38	
20	21.27	20.76	21.19	20.25	20.35	20.94	

SUMMARY							
	A.L.I	B.L.I	C.L.I	A+ L.I	B+ L.I	C+ L.I	
average	17.32	17.69	17.94	16.81	17.05	16.79	
modal	19	19	19	17	17-18	18	
best	18	17	18	17	17	17	
max expect	24.95	24.03	23.85	39.39	38.95	37.00	
	A.L.II	B.L.II	C.L.II	A+ L.II	B+ L.II	C+ L.II	
average	16.86	16.77	16.93	16.22	16.03	15.96	
modal	18	18	18	18	18	17	
best	18	18	18	17	17	17	
max expect	23.85	27.66	24.95	37.00	42.04	39.39	

Figure 7: Realized Payoffs and Summary Statistics

Our analysis of the results has been from the viewpoint of testing our theory, which is the main goal of the experiment. We discuss here some findings that are not directly relevant to our model but that are useful for a broader understanding of individuals' behavior.

In Figure 7 we report the realized payoffs for each action in the (modified) 11-20 game, computed using the empirical distributions observed in the various treatments for the two labels. Although distinct from the objectives of our theory, these realized payoffs allow for interesting observations. For instance, we find that the pure Nash equilibrium action in this game, 11, yields the lowest payoff in nearly all treatments, with optimal choices varying between 17 and 18. Therefore, a subject who has discovered the Nash equilibrium and plays accordingly would do worse (see Bosch-Domènech, Garcia-Montalvo, Nagel and Satorra (2002) for a discussion of this phenomenon). The rare occurrences of 11s and other low numbers in our data suggests that this kind of "curse of knowledge" is not particularly significant for the subjects of our experiment.

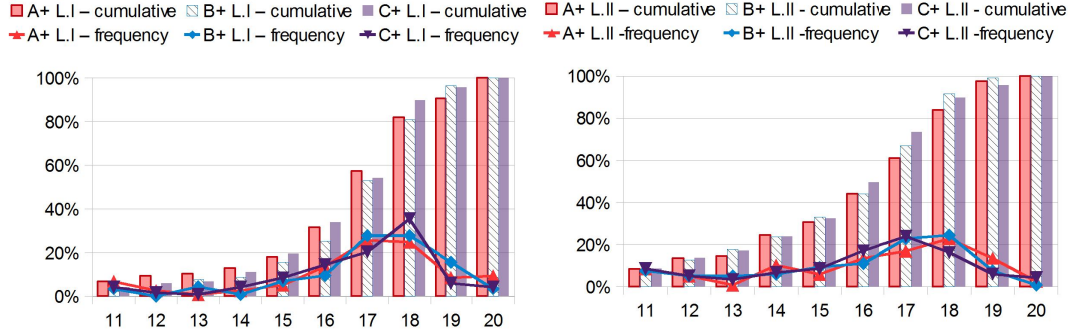


Figure 8: Treatments [A+], [B+] and [C+] for label *I* (left) and *II* (right).

We also report the transition matrices for the low-high payoff comparisons, which serve to track individuals' behavior across different treatments (Figure 6). Of particular interest in these matrices are the patterns of behavior concerning the choices of 20 and 11. Specifically, a plausible hypothesis is that 20 is chosen not only by level-0 players, but also by 'equilibrium players' who are not certain that the opponent would play 11 and have a strong degree of risk aversion (if the opponent plays 11, the payoffs from playing 11 and 20 is, respectively, 21 and 20). Observing large changes to and from 20 in the different treatments could be interpreted as evidence of this phenomenon.<sup>32</sup> This is not supported by the data, however, since transition matrices do not document a significant fraction of large changes to and from 20; the majority of these observations entail changes of two or three steps.<sup>33</sup>

Finally, we report the comparisons of the high payoff treatments [A+], [B+] and [C+]. From Figure 8, no discernible pattern emerges either for label *I* or for label *II*, and we note that the (frequency) distributions are close to each other. Viewed together with the results for treatments [A], [B] and [C] discussed in the main text, these results are indicative of label *I* subjects' beliefs. Specifically, they suggest that label *I* subjects believe that the cost functions associated with label *II* subjects are higher than their own at low levels of  $k$ , but become closer to their own cost function at higher  $k$ 's. In other words, label *I* subjects believe that, when sufficiently motivated, label *II* subjects are essentially the same as label *I*. An example of cost functions that satisfy this property is provided in Figure 2.a (p. 12). While the present analysis is not ideal to identify subjects' cost functions, an extension of our approach could be used for this purpose.

<sup>32</sup>We are grateful to one referee for this suggestion.

<sup>33</sup>We also note that the data from these matrices show that, at an individual level, roughly 80% of the observed changes are consistent with the theory.